Dynastic Accumulation of Wealth

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Abstract: Why do some dynasties maintain the fortune of their founders while others completely squander it in few generations? What are the causal mechanisms that underlie the intergenerational transmission of wealth? Is there a role for public policies aiming at improving long-run social mobility and wealth inequalities? To address these questions, we use a basic deterministic microfounded model based on two main ingredients: the "hunger for accumulation" and the "willingness to exert effort". The interplay between these two elements allows our dynamic model (*i*) to generate a variety of wealth accumulation and effort choice dynamics (*ii*) to provide an explanation for some macroeconomic features of social mobility and class structure as well as for the existence and the demise of the rich bourgeoisie. Furthermore, we analyze the effect on wealth accumulation of inheritance taxation and extend our setting to variable wage opportunities and exogenous shocks. Our analysis points to the crucial role of our two key ingredients, rather than of initial wealth or transitory shocks to wealth, in generating the patterns of wealth accumulation within a family lineage.

Keywords: Intergenerational accumulation, Social mobility, Wealth inequality, Spirit of capitalism, Effort choice, Inheritance taxation.

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1. Introduction

It is highly rare for a family fortune to last more than three generations. It is so much so that the well-known adage, often attributed to Andrew Carnegie, "Shirtsleeves to shirtsleeves in three generations" exists in different versions worldwide.[1] Its message is supported by a recent study by Cochell and Zeeb (2005) who found that 6 out of 10 families lose their fortunes by the end of the second generation and 9 out of 10 by the end of the third. Nevertheless, the idea of building a family legacy that lasts long after you are gone is not only seductive but possible. In fact there are many famous families, like the Rockefellers or the Rothschilds, that have built impressive patrimonies which have lasted, or are likely to last, more than 100 years. However, there is also an important proportion of families who do not receive any patrimony and/or may not be able to build one to start with.[2]

The main goal of this paper is to propose a theoretical framework that is able to explain the observed variety of wealth accumulation patterns within a family lineage (dynasty).[3] Why do some dynasties maintain the fortune of their founders while others completely squander it in few generations? What are the causal mechanisms that underlie the intergenerational transmission of wealth? Is there a role for public policies aiming at improving long-run social mobility and wealth inequalities? These are the questions we address.

The two key ingredients of the model we propose are the "hunger for accumulation" and the "willingness to exert effort". The first is a parameter related to Max Weber's theory of the "spirit of capitalism", wherein accumulating wealth has value in itself. This view has been shared by many other contemporary and classic economists, including A. Smith, J.S. Mill, J. Schumpeter and J.M. Keynes.[4] The inclusion of direct preferences over wealth typically takes the formulation of either "spirit of capitalism" or "pure joy-of-giving". Although these two specifications are, as it will be discussed later, extremely similar, the "spirit of capitalism" has been used by many authors who have tried to explain growth and/or savings (see, for instance, Bakshi and Chen 1996, Gong and Zou 2002, Zou 1994 and 1995, Carroll 2000, De Nardi 2004, Reiter 2004, and Galor and Moav 2006).

[1] Precisely this adage comes from an antique chinese proverb, "Rice paddy to rice paddy in 3 generations". The japanese, indian, and british versions are, respectively, "Kimono to kimono in 3 generations", "Sandals to sandals in 3 generations", and "Clogs to clogs in 3 generations".

[2] According to Kennickell (2006) only 20 % of families receives any inheritance (40 % in the top 5%).

[3] There is a vast empirical literature studying wealth and income mobility between two successive generations (see, e.g., Wahl (1995), Mulligan (1997), Solon (1999), and Charles and Hurst (2003)). However, due to inadequate statistical records on the evolution of wealth across several generations of the same family lineage there is little work on the analysis of long-run wealth dynamics within a dynasty. Arrondel and Grange (2006) provide a review of this literature as well as a study based on French data from 1800.

[4] See Zou (1992 and 1994) for a review of the history of economic thought and more references on this topic.

Consistent with empirical evidence, differently from "pure joy-of-giving", it generates the properties that the average propensity to bequest is an increasing function of wealth and that the wealth held by an individual does not always have an inherited component.

The second key ingredient is related to the introduction of a choice variable for effort which allows taking into account the predisposition towards working or, alternatively, the entrepreneurial[5] attitude. According to recent findings in psychology (see Bowles and Gintis 2002) people have different beliefs about the determinants of an individual's social status. Some think that rich are rich because of "hard work" and poor are poor because of "laziness". Others think that rich (poor) are rich (poor) because of (lack of) "luck" or "family money or connections". In our setting the willingness to exert effort will play a crucial role in the interaction between inherited wealth, effort (labor) supply, and transmitted wealth. The analysis of such interplay reveals in which circumstances these different beliefs can be justified.

At the beginning of the 20th century the liberal economist Franck Knight pointed out three main determinants of the accumulation and transmission of wealth: heritage, effort, and luck. Although our basic setting is based only on heritage and effort, a simple form of luck will be considered as an extension. Modeling luck typically requires the use of stochastic processes, which makes the analysis of the dynamics extremely complicated. In fact, works that follow this direction focus on the distribution of wealth in the steady-state. By abstracting from this form of luck we can explicitly study the dynamics of wealth accumulation. Consequently our approach can be viewed as a contribution complementary to both studies that calibrate stochastic growth models[6] and theoretical models with human capital accumulation[7] or imperfect credit markets.[8]

Our basic model considers the decisions of the members of a dynasty who each live one period and give birth to a child. For any given price of effort (wage) and the interest rate transforming end-of-period wealth into next generation's inherited wealth, each member of the dynasty chooses how much to consume, how much wealth to leave to the next generation, and whether to exert

[5]Entrepreneurial attitude is considered in the literature on family firms and dynastic management. See, for example, Caselli and Gennaioli (2006) and the literature therein.

[6]An elegant way to generate non-degenerate wealth distribution consists of introducing some idiosyncratic uninsurable risks in a growth model. Calibrated economies with Barro (1974)-Ramsey (1928) households have been, for example, studied by Aiyagari (1994), Castaneda, Diaz-Giménez and Rios-Rull (1998 and 2003) and Quadrini (2000), when agents have identical preferences or by Krusell and Smith (1998) when agents differ with respect to time discount rate.

[7]This literature has been initiated by Becker and Tomes (1979 and 1986), Loury (1981), and Benabou (1996).

[8]These standard theoretical models on wealth accumulation and on wealth distribution (see, for example, Banerjee and Newman (1991), Galor and Zeira (1993), Aghion and Bolton (1997), Piketty (1997), and Matsuyama (2000) and (2006)) generally have three main ingredients: imperfect capital market, exogenous prices and "pure joy of giving".

effort. We find that, for any given wage, there exists a level of wealth above which households decide not to exert effort. Similarly there exists a level of inherited wealth below which households decide to leave no wealth. The interplay between these two thresholds allows the model to generate, even in the absence of any source of uncertainty or capital market imperfections, a variety of long-run dynamics (hereafter LRD) of wealth and effort choice. For instance, in some circumstances a dynasty is caught into a poverty trap, wherein all generations work without leaving any wealth. Under some other circumstances, all members of a dynasty work but also transmit positive wealth. We also find situations were although no member works transmitted wealth, thanks to the high asset return, increases indefinitely over time. Interestingly, there are cases where there are successive runs of generations who do not work and squander part or the totality of the (finite) patrimony that was accumulated through the working effort of their predecessors.

We use our basic microfounded model to explain some macroeconomic features of social mobility and class structure as well as the existence and the demise of the rich bourgeoisie. In particular, we show that it provides a simple deterministic alternative to the sophisticated model of Matsuyama (2006) for the endogenous emergence of a stratified society, wherein inherently identical agents may endogenously separate into the rich and the poor. We also show that our model provides a possible interpretation for the demise of the rich bourgeoisie and the end of a class struggle, which is consistent with the recent explanations of Galor and Moav (2006) and Doepke and Zilibotti (2005 and 2008) rather than with those based on capital markets imperfections.

We also extend our basic setting to analyze the effect on wealth accumulation of inheritance taxation. Our study reveals that highly confiscatory taxes always induce people to work. As a consequence, paradoxically, high tax rates may lead to higher long-run average wealth than mild tax-rates or no tax. In addition, inheritance taxation may redistribute wealth and lifetime income intergenerationally.

Lastly, we relax some of the assumptions used in the basic setting by investigating the effect on the LRD of variable wage opportunities and of exogenous shocks. In particular we consider two types of variable wage opportunities. First, we let wage to grow at an exogenous fixed rate. Second, to determine the impact on wealth accumulation of the "family connections" mentioned earlier, we consider an "unequal opportunity" society, wherein the wage opportunities available to a generation depend on inherited wealth. These extensions give rise to new types of LRD but eliminate others. The introduction of exogenous shocks is the easiest way to introduce "luck" in our model. It allows us to analyze the reaction of the behavior of the dynasty to the existence of self-made men, such as, for example, Bill Gates. In this respect, we find that a big positive shock to wealth may paradoxically slow down the dynasty's long-run accumulation process.

An important final remark is that both the analysis of the basic model and of the different extensions point to the crucial role of the willingness to exert effort and the hunger for accumulation, rather than of initial wealth or transitory shocks to wealth, in generating the patterns of wealth accumulation within a family lineage.

The paper is organized as follows. Section 2 introduces the model. Section 3 is devoted to the characterization of the possible types of LRD. Section 4 considers the implications of our model with respect to the formation and dissolution of a stratified society as well as the existence and evolution of wealth inequalities. Section 5 considers the effect of a tax on inheritance on the LRD. Section 6 relaxes some of the modeling assumptions. Finally, Section 7 concludes. All proofs are gathered in the Appendix.

2. The Model

We consider one dynasty composed of successive generations of agents, each living one period and giving birth to a child. The problem faced by an agent, member of this dynasty, is the same in each period: given initial wealth, he has to decide whether to work, how much to consume, and how much to leave as end-of-the-period wealth.

Formally, we let $t = 0, ..., \infty$ denote the time. The agent at time t has initial (inherited) non-negative[9] wealth $X_t = R_t x_t \geq 0$, where $x_t \geq 0$ is[10] the wealth left by the previous generation and R_t is the asset gross rate of return. We consider a binary choice variable for[11] effort (labor), $e_t \in \{0, 1\}$, which takes the value 1 when the agent works and 0 otherwise. When the agent works he receives an exogenous wage w_t. The agent lifetime disposable income, $\Omega_t = w_t e_t + R_t x_t$, is allocated between consumption c_t and end-of-period wealth x_{t+1}.

2.1 - Preferences.

The preferences of an agent born in t are defined over his period consumption, c_t, his end-of-period wealth, x_{t+1}, which will be transferred to his offspring, and his effort level, e_t. Such preferences are represented by the following utility function:

$$U(c_t, x_{t+1}, e_t) = (1 - \beta) \ln c_t + \beta \ln(\varepsilon + x_{t+1}) - \xi e_t \tag{1}$$

[9]Negative bequests are not permitted. In most societies inherited debts are not enforceable. While governments can force future generations to reimburse debts contracted in benefit of past generations, families cannot.

[10]For simplicity there is only one risk-free asset in the economy and there is no explicit capital. Since each agent lives only one period and has to repay his debt within his lifetime, there is no active role for credit markets. Therefore, by default, our model can be thought of as a model without credit market imperfections.

[11]Inspired by Shapiro and Stiglitz (1984), this assumption is often used to microfoundate macroeconomic models (see, e.g., Piketty (1997) and most recently Matsuyama (2008)).

where $\beta \in (0,1)$, $\varepsilon \geq 0$ and $\xi \geq 0$.

Equation (1) is quite general, as it imbeds different specifications used in the literature. When $\xi = 0$ and $\varepsilon = 0$ we recognize the "pure joy of giving" (or "warm-glow") approach used by, for example, Galor and Zeira (1993), Aghion and Bolton (1997), and Piketty (1997). When $\varepsilon > 0$ and $\xi = 0$ we recognize the "spirit of capitalism" specification used by, for example, Carroll (2000), De Nardi (2004), and Galor and Moav (2006).

Our specification of the utility derived from wealth allows us to interpret the parameter β alternatively as a degree of dynastic altruism[12] or as the hunger for dynastic accumulation. Notice also that restricting to $\varepsilon > 0$ is equivalent to ruling out the condition of infinite marginal utility of wealth at $x_{t+1} = 0$. In addition, as it will be shown later, it allows having a richer set of dynamics of wealth accumulation as well as, consistent with empirical evidence, a saving rate which is increasing with lifetime wealth.[13] On this dimension our microfounded formulation is more empirically relevant than those of the standard literature on distributional dynamics with credit-rationing, where each agent leaves an exogenous fraction of his total income to the next generation (see, e.g., Piketty 1997, Aghion and Bolton 1997, or Matsuyama 2000).

As already pointed out, one of the novelties of this paper is the introduction of effort. The parameter ξ represents the willingness to exert effort or, in other words, the cost of effort. It is a key parameter in our setting, as it drives the response of effort choice to wealth. Such response will, in turn, determine the different typologies of dynastic wealth dynamics. While everybody would agree that such parameter is (at least weakly) positive, its magnitude is controversial. In fact, empirically it is not clear what the effect of wealth on labor force participation (in our case effort) is.[14] Our theoretical model allows analyzing the possible effects of wealth on effort and wealth transmission as a function of ξ.[15]

[12] Michel, Thibault, and Vidal (2006) provide a review of the intergenerational altruism literature.

[13] As noted by Galor and Moav (2006): "This utility function represents preferences under which the saving rate is an increasing function of wealth. This classical feature (see, e.g., Keynes (1920), Lewis (1954), and Kaldor (1957)) is consistent with empirical evidence. Dynan, Skinner and Zeldes (2000) find a strong positive relationship between personal saving rates and lifetime income in the United States. They argue that their findings are consistent with models in which precautionary saving and bequest motives drive variations in saving rates across income groups. Furthermore, Tomes (1981) and Menchik and David (1983) find evidence that the marginal propensity to bequeath increases with wealth."

[14] It is difficult to estimate such effects when changes in wealth are expected, small, or happen early in life. See, e.g., Poterba (2000).

[15] Evidence about a positive elasticity of leisure to wealth is documented by the fact that retirement decisions are positively related to unforseen wealth shocks, like, for example, lotteries (see Poterba (2000)) and inherited wealth higher than expected (see Brown, Coile, and Weisbenner (2006)). In our model agents live only one period. One

2.2 - Optimal choices of effort and wealth transmission.

The following two propositions characterize the optimal choices of effort and wealth transmission of an agent living in t.

Proposition 1 *An agent living in t transmits to his child an increasing proportion of his disposable income Ω_t, which is independent of prices w_t and R_t. In particular:*

$$x_{t+1} = \begin{cases} 0 & if \quad \Omega_t \leq \sigma\varepsilon \\ \beta\Omega_t - (1-\beta)\varepsilon & if \quad \Omega_t > \sigma\varepsilon \end{cases}, \tag{2}$$

where $\sigma = 1/\beta - 1$.

Proof - See Appendix A.

Proposition 1 tells us that the agent leaves a positive wealth only when lifetime income is sufficiently high. Otherwise an agent may be captured into a poverty trap, wherein he is so poor (inherited wealth and wage are both very low) that he consumes all of his resources without leaving any wealth to his successor. Because in our model the propensity to save can be defined as the ratio $x_{t+1}/\ \Omega_t$, Proposition 1 tells us that, coherent with empirical evidence, the propensity to save is zero for individuals with low lifetime income and is increasing in lifetime income otherwise. Notice also that, the higher the hunger for accumulation β the lower the threshold for transmitting positive wealth $\sigma\varepsilon$ and the higher transmitted wealth x_{t+1}.

For what concerns the effort choice, while it is obvious that when an agent inherits no wealth he decides to work (otherwise he would have zero consumption), when inherited wealth is positive he may decide not to work.

Proposition 2 *There exists a positive threshold $\mathcal{X}_t$, increasing in w_t but independent of R_t, such that an agent living in t decides not to exert effort if and only if his inherited wealth $X_t = R_t x_t$ is greater than $\mathcal{X}_t$. Hence:*

$$e_t = \begin{cases} 1 & if \quad X_t \leq \mathcal{X}_t \\ 0 & if \quad X_t > \mathcal{X}_t \end{cases}.$$

Proof - See Appendix B.

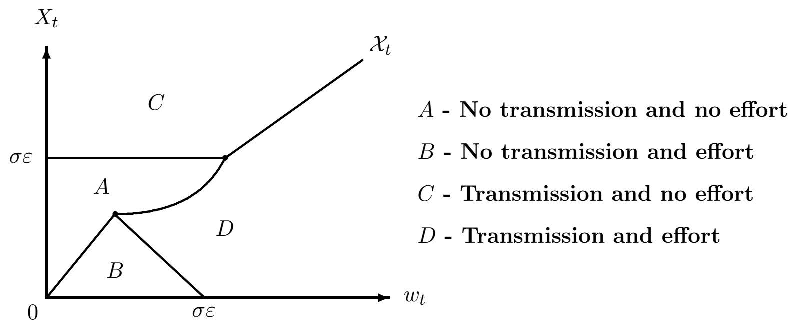
Let $\mu_1 = (1-\beta)\left(1 - 1/e^{\xi/(1-\beta)}\right)/\beta$ and $\mu_2 = \left(e^{\xi} - 1\right)/\beta$, where $\mu_1 < \sigma$ and $\mu_1 < \mu_2$. As shown in Appendix B, there exist three thresholds $X_t^{\sharp}$, $X_t^{\star}$ and $\overline{X}$ satisfying $X_t^{\sharp} \leq \sigma\varepsilon - w_t \leq X_t^{\star} \leq \sigma\varepsilon \leq \overline{X}_t$

could reinterpret the individual problem as the decision he has to make as whether to go on early retirement as a response to an unexpected inheritance.

such that:

$$\mathcal{X}_t = \begin{cases} X_t^{\sharp} & \text{if} \quad w_t \leq \mu_1 \varepsilon \\ X_t^{\star} & \text{if} \quad \mu_1 \varepsilon < w_t < \mu_2 \varepsilon \\ \overline{X}_t & \text{if} \quad w_t \geq \mu_2 \varepsilon \end{cases} .$$

Figure 1 provides a very useful graphical representation of the optimal decisions about whether to transmit a positive wealth and/or to work as a function of the wage and inherited wealth, as implied by Propositions 1 and 2.



– Figure 1 –

Above the line[16] $\mathcal{X}_t$, that is in regions A and C, an agent living in t decides not to work. The decision about wealth transmission x_{t+1} is determined by the line $X_t = \sigma\varepsilon$, above which (region C) the agent transmits positive wealth and below which (region A) he leaves zero wealth. Below the threshold $\mathcal{X}_t$, that is in regions B and D, the agent decides to work. In these regions the decision about x_{t+1} is determined by the line $X_t = \sigma\varepsilon - w_t$. To its left (region B) the agent leaves no wealth and to the right (region D) he leaves positive wealth.

From the above results it follows that transmitted wealth x_{t+1} may be non monotonic in inherited wealth X_t. For instance for wages between $\mu_1\varepsilon$ and $\sigma\varepsilon$, when inherited wealth is zero (region B), the agent leaves no bequest to his successor. As the level of initial wealth X_t increases, eventually we reach region D, where the agent leaves positive wealth. However, as initial wealth increases further we eventually enter into region A, where the agent goes back to transmitting no wealth.

It is important to notice that when $\varepsilon = 0$, the relevant part of Figure 1 becomes the upper right region ($\sigma\varepsilon$ collapses to zero), where only regions C and D exist. That is, in presence of pure *joy-of-giving* agents always leave a positive wealth to the successive generation. Such a simpler formulation although more tractable does not take into consideration the empirical evidence about the existence of a large proportion of (poor) agents who do not leave positive bequests because their lifetime income is barely enough to finance their own consumption.

[16] This line is increasing. It is linear for low and high wages and it is convex for intermediate wages.

3. Dynamics of dynastic wealth

In this section we characterize the full dynamics of wealth from generation to generation within a dynasty. As common in the closely related literature, for simplicity we assume that R_t and w_t are constant through time[17].[18] A straightforward application of Propositions 1 and 2 leads to the following dynamic equation of wealth.

Proposition 3 *The evolution of dynastic wealth is given by the following dynamic equation:*

$$X_{t+1} = \begin{cases} 0 & \text{if} \quad X_t \leq \sigma\varepsilon - w \text{ or } X^* < X_t \leq \sigma\varepsilon \\ \beta R[w + X_t - \sigma\varepsilon] & \text{if} \quad \sigma\varepsilon - w < X_t \leq X^* \text{ or } \sigma\varepsilon < X_t \leq \overline{X} \\ \beta R[X_t - \sigma\varepsilon] & \text{if} \quad X_t > \overline{X} \end{cases} . \tag{3}$$

The dynamic equation of dynastic wealth is determined by three branches. In the first branch wealth is equal to zero. In the other two branches the evolution of wealth is defined by an arithmetic-geometric progression which depends on whether the agent is exerting effort. Proposition 3 provides a compact description of the evolution of wealth. The reader can find its complete characterization as a function of wage in Appendix C.

3.1 - Typologies and properties of long-run dynamics.

Since our model is deterministic, for any given initial wealth X_0 we can trace all the sequence, X_1, X_2, ... of dynastic wealth. Although initial wealth X_0 might affect the behavior of the first generations, starting from a certain period which is not necessarily too far away, we can find some regularities in the behavior of future generations. We have to clarify this point because in general the evolution of wealth and work behavior across members of the same dynasty does not necessarily

[17] As noted by Davies and Shorrocks (1999) simple models of wealth accumulation assume "that everyone faces the same constant rate of return, r". Two usual ways are used to macrofound this assumption. As Galor and Zeira (1993) we can consider a small open economy in which production is described by $Y_t = F(K_t, L_t)$ where Y_t is output at period t, K_t is the amount of capital and L_t is labor input. F is a concave production function with constant returns to scale. Investment in capital is made one period in advance. Capital is perfectly mobile. "The world rate of interest is equal to R and is assumed to be constant over time." Since $F'_K(K_t, L_t) = R$, "there is a constant capital-labor ratio, which determines the wage of labor w which is constant as well." Alternatively, we can assume, as Dutta and Michel (1998), that the production technology is linear: $Y_t = \rho K_t + wL_t$. Production is carried out by competitive firms, which maximize profits. It follows that the real wage rate $w_t = w$ at each period t. Capital depreciates at rate δ each period so that $K_{t+1} = (1+\delta)K_t + I_t$, where I_t is the level of investment in each period. Firms choose capital to maximize operating profits plus the resale value of the firm. In this context $\delta + r_t = \rho \equiv R$.

[18] As Galor and Zeira (1993) we relax in Section 6 the standard assumption of constant wages and consider the case where wages grow at a constant rate. According to Kaldor, who summarizes into stylized facts a number of empirical regularities in the growth process in industrialized countries, the rate of return to capital R is almost constant over time while the wage w grows at a positive constant rate.

exhibit stationary patterns in our model. The characterization of the above mentioned regularities, which we refer to as "long-run dynamics", is the object of this section.

Before defining and formally characterizing the types of LRD that can emerge in our setting, some clarifications about the terminology that we will use are in order. We break-down the class of individuals who inherit a positive wealth and do not work into three sub-categories. We define as: (a) *rentiers* those agents who do not work but nevertheless transmit a level of wealth greater than the wealth they have inherited; (b) *dilapidators* those agents who do not work and transmit a (strictly positive) level of wealth which is lower than the one they have inherited; and (c) *ruiners* those agents who receive positive wealth but neither work or leave positive wealth.

The general typologies of LRD that can be observed in a society and their properties are summarized in Table 1. To each type of LRD we assign a composite name composed of two parts. The first indicates the type of long-run wealth that the LRD allows reaching. It is ZERO if dynastic wealth converges towards or periodically becomes zero; FINI if it converges towards a finite value, and INFI if it grows unboundedly. The second indicates the long-run working status of the dynasty. It is WORK if in the long run each member of the dynasty works, RENT if in the long run no member of the dynasty works, and MIX if there exists a mix of workers and non-workers (*dilapidators* or *rentiers*). It follows that a LRD is said to be:

- ZERO-WORK if, from a certain period, all generations work and choose not to transmit any wealth.
- FINI-WORK if, from a certain period, all generations work and transmitted wealth monotonically converges towards a positive finite value.
- INFI-WORK if, from a certain period, all generations work and transmitted wealth monotonically increases towards infinity.
- ZERO-MIX if, from a certain period, there is periodically one generation who does not work and completely squanders all the wealth accumulated by the preceding generations.
- FINI-MIX if, from a certain period, there are infinite successive runs of generations who work and build up an upper-bounded patrimony and of generations who do not work and squander part of their initial wealth.
- INFI-MIX if, from a certain period, there are infinite successive runs of generations who work and build up a patrimony which tends to infinity and of generations who do not work and squander part of their initial wealth.
- INFI-RENT if, from a certain period, no generation works and transmitted wealth increases monotonically towards infinity.

Type of LRD	Long-run wealth	Dynamics of accumulation	Long-run existence of			
			workers	dilapidators	ruiners	rentiers
ZERO-WORK	Zero	Constant	Yes	No	No	No
FINI-WORK	Finite	Monotone	Yes	No	No	No
INFI-WORK	Infinite	Monotone	Yes	No	No	No
ZERO-MIX	Zero/Finite	Cyclical	Yes	Possible	Yes	No
FINI-MIX	Finite	Cyclical	Yes	Yes	No	No
INFI-MIX	Infinite	Cyclical	Yes	Yes	No	No
INFI-RENT	Infinite	Monotone	No	No	No	Yes

- Table 1 -

In a ZERO-WORK LRD, the dynasty is caught into a poverty trap, where in the long-run all generations work and consume all of their wage without transmitting any wealth. In a FINI-WORK or INFI-WORK LRD all generations work but wealth from generation to generation either converges monotonically to a positive finite value $\widetilde{X}$ (in the case of a FINI-WORK LRD) or increases monotonically up to becoming asymptotically infinite (in the case of an INFI-WORK LRD). An interesting feature of the MIX types of LRD is that wealth fluctuates. In a ZERO-MIX LRD the cycles are regular and there exists periodically one generation, the *ruiner*, who completely squanders all the wealth. This *ruiner* could appear after a sequence of generations who cumulated (an increasing) positive wealth or could appear after a sequence of *dilapidators*. Both in the FINI-MIX and the INFI-MIX LRD the cycles need not to be regular and wealth is always bounded away from zero, implying the existence of *dilapidators* but the absence of *ruiners*.

Notice that our setting can generate different degrees of inter-generational mobility. This is characterized by the changes in the working and wealth accumulation behaviors among members of the same dynasty in correspondence to the different LRD. In 4 out of 7 LRD the status of the members of a dynasty is invariant in the long run. However, mobility exists in all the three LRD of type MIX.

3.2 - Characterizations of the long-run dynamics.

The determination of the LRD of an economy hinges on the analysis of the non-trivial branches of the dynamic equation (3). For instance, it is important to notice that when the wage is sufficiently appealing (i.e., $w > \sigma\varepsilon$), the wealth accumulated by an infinite sequence of workers tends towards infinity if $\beta R > 1$ and towards $\widetilde{X} = \beta R[w - \sigma\varepsilon]/(1 - \beta R)$ if $\beta R < 1$. When $\beta R < 1$, an agent deciding not to work always transmits a level of wealth which is lower than the one he had inherited. It is this type of decumulation of wealth that gives rise to a succession of generations who decide not to work and dilapidate part or the integrality of a given inheritance. Conversely, when $\beta R > 1$, an agent deciding not to work transmits a level of wealth greater than the one he had inherited if and only if the latter is greater than $\widehat{X} = \beta R\sigma\varepsilon/(\beta R - 1)$. It is this type of wealth accumulation that makes the emergence of *rentiers* possible.

Using these results and letting $\{\Delta_{X_0}(t)\}_{t=0}^{\infty}$ denote the complete trajectory of wealth when $w > \max\{\mu_2\varepsilon, \sigma\varepsilon\}$ and initial wealth is X_0 (see Appendix C) we now characterize the types of LRD generated by our model. We distinguish such characterization according to the relative values of the interest rate and the hunger for accumulation, i.e. $\beta R < 1$ or $\beta R > 1$.

Proposition 4 *When $\beta R < 1$ the LRD is:*
- ZERO-WORK *if and only if* $w \leq \sigma\varepsilon$.
- FINI-WORK *if and only if* $\left[\sigma\varepsilon < w < \mu_2\varepsilon \text{ and } \tilde{X} < X^\star\right]$ *or* $\left[w \geq \max\{\sigma\varepsilon, \mu_2\varepsilon\} \text{ and } \tilde{X} < \overline{X}\right]$.
- ZERO-MIX *if and only if* $\left[\sigma\varepsilon < w < \mu_2\varepsilon \text{ and } X^\star < \widetilde{X} < \sigma\varepsilon\right]$ *or* $\Big[\sigma\varepsilon < w < \mu_2\varepsilon,\ \sigma\varepsilon < \widetilde{X}$ *and* $\exists\, t, t'$ *such that* $\Delta_{X_0}(t)$ *and* $\Delta_0(t') \in (X^\star, \sigma\varepsilon)\Big]$.
- FINI-MIX *if and only if* $\left[\sigma\varepsilon < w < \mu_2\varepsilon,\ \sigma\varepsilon < \widetilde{X} \text{ and } \forall t > 0\ \Delta_{X_0}(t) \text{ or } \Delta_0(t) \in (0, X^\star) \cup (\sigma\varepsilon, +\infty)\right]$ *or* $\left[w \geq \max\{\sigma\varepsilon, \mu_2\varepsilon\} \text{ and } \overline{X} < \widetilde{X}\right]$.

PROOF – See Appendix D.

The intuition behind the results of Proposition 4 is the following. When the wage is at a subsistence level ($w \leq \sigma\varepsilon$), eventually all members of the dynasty choose to work and to not transmit any wealth. Clearly this is the case when inherited wealth is small, as the agent is forced to work and to allocate income completely to consumption. Interestingly, this pattern holds for any level of initial wealth. In fact when initial wealth is sufficiently high ($X_0 > \sigma\varepsilon$), due to the low wage, the initial generation and potentially some of the successive ones decide not to work and to finance consumption with inherited wealth. Therefore, in a finite time the latter becomes zero and stays zero thereafter. This ZERO-WORK LRD entails a poverty trap and is consistent with the empirical evidence that consumption tracks current income (see Wolff, 1998) and that many households (80% according to Kennickell, 2006) do not receive an inheritance.

For higher wages ($w > \sigma\varepsilon$) the LRD can be of three types: FINI-WORK, FINI-MIX or ZERO-MIX. In order for a FINI-WORK LRD to exist, the limiting value $\widetilde{X}$ of wealth accumulated by an infinite sequence of workers must be no greater than the threshold $\mathcal{X}$ above which individuals choose not to work. When $X_0 < \mathcal{X}$ all generations work and wealth converges to $\widetilde{X}$ monotonically from below. When $X_0 > \mathcal{X}$ the initial generations decide not to work and decumulate wealth. However, once $\widetilde{X} < X_t < \mathcal{X}$, all future generations work and wealth monotonically decreases towards $\widetilde{X}$.

The LRD is of type MIX whenever $\widetilde{X}$ is greater than $\mathcal{X}$. This is because, once there is a member of the dynasty who does not work (there is always some when $\widetilde{X} > \mathcal{X}$), transmitted wealth becomes lower than inherited wealth. Wealth decreases through time until it becomes too low for the following generation to decide not to work. Of the two types of MIX LRD a ZERO-MIX could exists only when $\sigma\varepsilon < w < \mu_2\varepsilon$. In fact the wage w must be greater than $\sigma\varepsilon$, otherwise we would have a ZERO-WORK LRD, and lower than $\mu_2\varepsilon$, otherwise transmitted wealth could never become zero. A FINI-MIX LRD exists only if once wealth becomes positive it never goes back to zero. This is clearly the case when $w > \max\{\sigma\varepsilon, \mu_2\varepsilon\}$ and it could happen also when $\sigma\varepsilon < w < \mu_2\varepsilon$.

It should be noticed that all types of LRD obtained when $\beta R < 1$ (except when, as shown in Appendix D, $\sigma\varepsilon < w < \mu_2\varepsilon$ and $\widetilde{X} > \sigma\varepsilon > \beta R[w + X^* - \sigma\varepsilon]$) are robust, in that they do not depend on initial wealth X_0. We now characterize the LRD when $\beta R > 1$.

Proposition 5 *When $\beta R > 1$ the LRD is:*

- ZERO-WORK *if and only if $w \leq \sigma\varepsilon$ and there exists a period T such that $X_T = 0$.*
- ZERO-MIX *if and only if $[\sigma\varepsilon < w < \mu_2\varepsilon$ and $\exists\ t, t'$ such that $\Delta_{X_0}(t)$ and $\Delta_{X0}(t') \in (X^\star, \sigma\varepsilon)]$.*
- FINI-MIX *if and only if $\Big[\mu_1\varepsilon < w < \min\{\sigma\varepsilon, \mu_2\varepsilon\}$ and $\forall t \geq 0\ \Delta_{X_0}(t) \in (\max\{\sigma\varepsilon - w, \widetilde{X}\}, X^\star) \cup(\sigma\varepsilon, \widehat{X})\Big]$ or $\Big[\mu_2\varepsilon < w < \sigma\varepsilon$ and $\forall t \geq 0\ \Delta_{X_0}(t) \in (\max\{\sigma\varepsilon - w, \widetilde{X}\}, \widehat{X})\Big]$ or $[\sigma\varepsilon < w < \mu_2\varepsilon$ and $\forall t \geq 0,\ \Delta_{X_0}(t) \in (0, X^\star) \cup (\sigma\varepsilon, \widehat{X})\Big]$ or $\Big[w \geq \max\{\sigma\varepsilon, \mu_2\varepsilon\}$ and $\forall t \geq 0 \Delta_{X_0}(t) \leq \widehat{X}\Big]$.*
- INFI-RENT *if and only if there exists a period T such that $X_T > \widehat{X}$.*

PROOF – See Appendix D.

Different from the previous case, the types of LRD obtained when $\beta R > 1$ always depend on the level of initial wealth, X_0. When the wage is relatively low ($w \leq \mu_1\varepsilon$) a ZERO-WORK LRD emerges for low levels of inherited wealth. This is because, although for each dollar bequeathed the next generation receives more than 1 dollar, the level of wealth is too low to consider leaving a big portion of it to the next generation. Transmitted wealth decreases over time and eventually becomes zero. Conversely, an INFI-RENT LRD is obtained for high levels of inherited wealth. All members of the dynasty will choose not to work and nevertheless, because of the high interest rate and/or hunger for accumulation, will leave increasing bequests. When $\mu_1\varepsilon < w \leq \sigma\varepsilon$, in addition to the ZERO-WORK and to the INFI-RENT LRD it is also possible to have a FINI-MIX LRD. When $w > \max\{\sigma\varepsilon, \mu_2\varepsilon\}$ we find the same type of MIX LRD as with $\beta R < 1$. In addition, while with $\beta R < 1$ it was only possible to have a FINI-WORK LRD, with $\beta R > 1$ it is only possible to have INFI-RENT LRD.

Table 2 summarizes the results of Propositions 4 and 5. The LRD in the table that are underlined correspond to those not obtainable starting from $X_0 = 0$. At this point we have a wider understanding of the role played by the parameter ε. Our formulation ($\varepsilon > 0$) generates five possible typologies of wealth dynamics. Conversely, considering *pure joy-of-giving* ($\varepsilon = 0$) would deliver only three of them (a FINI-WORK or a FINI-MIX when $\beta R < 1$ and an INFI-RENT when $\beta R > 1$).[19]

	$\beta R < 1$	$\beta R > 1$
$w \leq \mu_1 \varepsilon$	ZERO-WORK	ZERO-WORK INFI-RENT
$\mu_1 \varepsilon < w \leq \sigma \varepsilon$	ZERO-WORK	ZERO-WORK FINI-MIX INFI-RENT
$\sigma \varepsilon < w \leq \mu_2 \varepsilon$	FINI-WORK FINI-MIX ZERO-MIX	FINI-MIX ZERO-MIX INFI-RENT
$w > max\{\sigma\varepsilon, \mu_2\varepsilon\}$	FINI-WORK FINI-MIX	INFI-RENT FINI-MIX

– Table 2 –

A graphical representation of the results of Propositions 4 and 5 makes the study of the role of prices (w and R) and of the effort parameter (ξ) easier and will be at the basis of the comparative static results implicit in the analysis of the next sections. It also allows pointing out the ability of our framework to explain part of the labor choices and wealth dynamics observed in our contemporary societies. However, due to the many configurations that can emerge in equilibrium and in order not to distract the attention of the reader from the most relevant results, the tedious derivation of such a graphical treatment is provided in Appendix E.

4. Class Structure and Inequality

Our basic microfounded model is able to explain some macroeconomic features of social mobility and class structure as well as the existence and the demise of the rich bourgeoisie. In the first subsection we show that our model provides a simple deterministic alternative to the sophisticated model of Matsuyama (2006) for the endogenous emergence of a stratified society, wherein inherently identical agents may endogenously separate into the rich and the poor. As Matsuyama (2006), for some parameter values, our simple economy predicts the rise of class societies.

In the second subsection we use our model to provide a possible interpretation for the demise of the rich bourgeoisie and the end of a class struggle, which is consistent with the recent explanations by Galor and Moav (2006) and Doepke and Zilibotti (2005 and 2008) rather than with those based on capital markets imperfections.

[19]In fact, when $\varepsilon = 0$ the possibility of a FINI-MIX LRD in the case represented in the last row of Table 2 vanishes, as when $\varepsilon = 0$ automatically $\widehat{X} = 0$.

4.1 - A simple model of endogenous class society.

In our model the social class attained by a dynasty (characterized by the long-run working behavior and pattern of wealth) depends in big part on the hunger for accumulation. We study the role of the parameter β on the possibility of social stratification and on the evolution of inequality.[20]

We consider two dynasties, indexed by a and b, with zero initial wealth and different hunger for accumulation, $\beta_a > \beta_b$. We focus only on relatively high wages, that is $w > \max\{\sigma\varepsilon, \mu_2\varepsilon, (R-1)\varepsilon\}$. Since both dynasties start with zero initial wealth, the first member of each dynasty works. However, in the long run the working behavior of the two dynasties may differ, depending on their hunger for accumulation relative to the two thresholds β_1 and β_2, where $\beta_1 = [R\varepsilon + \overline{X}]/[R(w+\varepsilon+\overline{X})] < 1/R$ corresponds[21] to the solution of $\overline{X} = \widetilde{X}$ and $\beta_2 \geq 1/R$ corresponds to the minimum β such that starting from $X_0 = 0$ it is possible to converge to an INFI-RENT LRD.

Hunger β_b and β_a $(\beta_b < \beta_a)$	LRD of Dynasty b / Dynasty a
$\beta_b < \beta_a < \beta_1$	FINI-WORK / FINI-WORK
$\beta_b < \beta_1 < \beta_a < \beta_2$	FINI-WORK / FINI-MIX
$\beta_b < \beta_1 < \beta_2 < \beta_a$	FINI-WORK / INFI-RENT
$\beta_1 < \beta_b < \beta_a < \beta_2$	FINI-MIX / FINI-MIX
$\beta_1 < \beta_b < \beta_2 < \beta_a$	FINI-MIX / INFI-RENT
$\beta_2 < \beta_b < \beta_a$	INFI-RENT / INFI-RENT

- Table 3 -

Table 3 summarizes the social class reached by each of the two dynasties in the long run for any possible value of their hunger for accumulation. First, notice that whenever the hunger for accumulation is greater than β_1, some or all members of a dynasty will not belong to the working class (as it is instead the case in a FINI-WORK LRD). Table 3 also shows that in three out of six configurations there is an endogenous emergence of social stratification. In fact, although both dynasties start with the same initial wealth and wage opportunities, due to a different hunger for accumulation they end up in different social classes. The striking result is that such endogenous stratification is possible even when the two dynasties are almost identical (i.e., the differences in their hunger for accumulation is infinitesimal).[22]

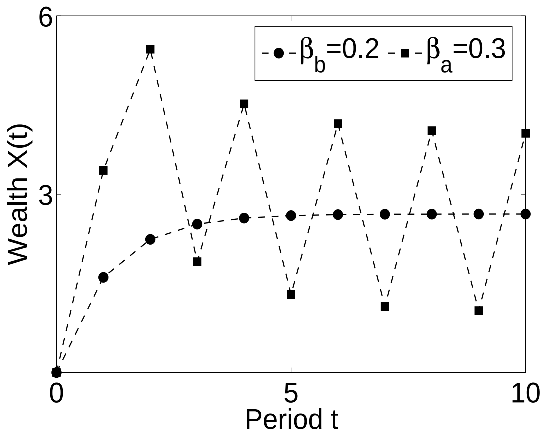
We now turn the attention to the evolution of wealth inequality between the two dynasties.

[20] Obviously, dynasties can be heterogeneous with respect to other characteristics. As long as heterogeneity concerns the effort cost ξ and/or the wage w, the results derived in Appendix E and summarized in Figures 10-15 can be directly used to infer the type of social stratification that can emerge.

[21] The assumption that $w > (R-1)\varepsilon$ guarantees that $\beta_1 < 1/R$.

[22] Considering $\epsilon \sim 0$, this is the case if $\beta_a = \beta_1 - \epsilon/2$ and $\beta_b = \beta_1 + \epsilon/2$ or if $\beta_a = \beta_2 - \epsilon/2$ and $\beta_b = \beta_2 + \epsilon/2$.

Because the wealth accumulated by the first dynasty a is higher than the one accumulated by the first generation of dynasty b, wealth inequality always emerges in the short run. If none of the dynasties ever exits the working-class, in the long run wealth inequality increases towards the finite value $\widetilde{X}_a - \widetilde{X}_b$. When at least one of the two dynasties has an INFI-RENT LRD, inequality can indefinitely increase in the long run.



- Figure 2 -

Interestingly there exist configurations where, even without the intervention of the government, inequalities are temporarily reduced. This can be the case when the LRD of dynasty a (with the greatest hunger for accumulation) is of type FINI-MIX and the dilapidators of dynasty a appear during a period where the members of dynasty b work and accumulate wealth. This case is depicted in Figure 2, for values of parameters $\varepsilon = 1$, $\xi = 1$, $w = 8$ and $R = 2$.

4.2 - A theory of the demise of the rich bourgeoisie.

We now use our framework to explain the existence and the demise of the 19th century's European class structure: the rich bourgeoisie and the poor proletariat. To this purpose, let us consider an economy with a continuum of dynasties, heterogeneous with respect to initial wealth X_0, effort cost ξ, and hunger for accumulation β.

A possible explanation for the existence of a capitalist-workers class structure before the industrial revolution could be the prevalence of very low "subsistence" wages. In this case, corresponding in our model to $w < \mu_1 \varepsilon$, society is inevitably divided into two classes (see Table 2): the poor workers and the rich rentiers (or capitalists) with a ZERO-WORK and an INFI-RENT LRD, respectively. The first class includes agents with low initial wealth (i.e., $X_0 < \widehat{X}$) and those with low hunger for accumulation (i.e., $\beta < 1/R$). The second class includes agents who have both a sufficiently high hunger for accumulation and high initial endowment (i.e., $\beta > 1/R$ and $X_0 > \widehat{X}$). In this society class mobility is inexistent, as the two possible LRD depend on initial wealth and in a ZERO-WORK LRD wealth never grows, and wealth inequalities exacerbate over time.

The presence of relatively high wages during the 20th century could explain, consistent with the theory of Galor and Moav (2006), why a capitalist-worker class structure did not survive.[23] In fact when wages are high $w > \max\{\sigma\varepsilon, \mu_2\varepsilon\}$, there are three social classes in our economy (see Table 2) characterized by dynasties with FINI-WORK, FINI-MIX, and INFI-RENT LRD. In this new society agents are richer, as not only the wage is higher but inherited wealth is always positive in the long run. For the same reason, economic differences between the rich rentiers and the other social classes are reduced. Furthermore, initial endowment is less important in determining the evolution of wealth. In fact even dynasties with zero or little initial wealth can, in the long run and depending on their hunger for accumulation, belong to any social class. As a consequence, in the medium run social mobility is higher and we are likely to observe new capitalists emerging from the middle and lower classes.

Our model can therefore account for both the passage from a two-class very unequal society to a three-class less unequal society and the decreasing importance over time of initial wealth (and, hence, for example, of landowners) just by considering the wage increase observed in the 20th century.

Notice that in our setting extremely wealthy agents (see Proposition 2) as well as agents with high cost of effort (see Figures $10 - 12$ in Appendix E) will choose not to work (or innovate). It follows that the prediction of our model are also consistent, under appropriate assumptions on the correlation between wealth and the cost of effort, with Doepke and Zilibotti (2005 and 2008). According to them, the decline of the bourgeoisie is the result of the endogenous choice of the rate of time impatience, which conducts the middle class to become patient and more willing to engage in new costly but profitable opportunities.[24] Different from models with capital market

[23]According to Galor and Moav (2006) during the 20th century we assisted to a wage increase because "The capitalists found it beneficial to support universal publicly financed education, which enhanced the participation of the working class in the process of human and physical capital accumulation, and led to a widening of the middle class and to the eventual demise of the capitalists-workers class structure". While wage is exogenous in our model, it delivers implications coherent with this theory. We will partially endogenise wages in Section 6 by introducing the hypothesis of "unequal opportunities".

[24]They focus on the puzzle of the rapid decline of the bourgeoisie: "why did the upper classes prove unable to exploit the new opportunities arising with industrialization, in spite of their superior wealth and education? Economic theories of wealth inequality often appeal to capital market imperfections: poor individuals may be unable to finance otherwise profitable investment projects, and are therefore forced to enter less productive professions. According to this theory, when new technological opportunities arise, the rich (who are least constrained by credit market imperfections) should be the first beneficiaries. Indeed, this theory should be highly relevant for the British Industrial Revolution, because wealth inequality was quite extreme and financial markets shallow by modern standards. Yet, we know now that the old rich did not do well at all, and were overtaken by a new economic elite that rose from the middle classes."

imperfections (see, e.g., Banerjee and Newman 1993, Galor and Zeira 1993 or Matsuyama 2006), our model predicts that those who take the effort to innovate and take advantage of new profitable opportunities are agents who are neither too poor nor too rich.

We acknowledge that in many countries the disappearance of rentiers has been attributed to the implementation of taxes on wealth/capital and/or to the introduction of redistributive policies (see, e.g., Piketty and Saez 2003, or Kopczuk and Saez 2004). We reserve the treatment of taxation to the next section.

5. Taxation

In this section we extend the basic setting of Section 3 to incorporate a tax on inheritance.[25] To this purpose we consider, as in Section 4.1, an economy composed of two types of homogeneous dynasties: a and b, in proportion p_a and $1 - p_a$, respectively. Members of dynasty a care about their end-of-period wealth (i.e., $\beta_a > 0$) while members of dynasty b do not (i.e., $\beta_b = 0$). One could also interpret these two types as being altruistic and egoistic respectively (see Mankiw 2000). We assume that at the beginning of each period t the government taxes inheritance at a rate $\tau \in [0, 1]$ and distributes back the corresponding revenues uniformly to all individuals. Letting θ_t denote the (lump-sum) transfer received by an individual at time t, we can write the budget constraint of the government as $\theta_t = \tau p_a R x_t^a$, where x_t^a is the wealth accumulated by a member of a dynasty of type a living in period $t - 1$.

Since we are interested in the effect of taxation on the dynamics of wealth accumulation, without loss of generality we focus on the behavior of dynasty a only.[26] A member of dynasty a maximizes his utility $U^a(c_t^a, x_{t+1}^a, e_t^a) = (1 - \beta) \ln c_t^a + \beta \ln(\varepsilon + x_{t+1}^a) - \xi e_t^a$, subject to his budget constraint $\Omega_t^a = w_t e_t^a + (1 - \tau) R x_t^a + \theta_t$.

Lifetime income of the member of dynasty a at time t, Ω_t^a, can be decomposed into two parts: a first part, $w_t e_t^a$, deriving from the effort exerted by the member of the dynasty during time t, and a second part, $(1 - \tau) R x_t^a + \theta_t$, deriving from the resources he receives at the beginning of time t. We can therefore reinterpret the second term as inherited wealth in the context of taxation and let $X_t^\tau = (1 - \tau) R x_t^a + \theta_t = R^\tau x_t^a$, where $R^\tau = \eta^a R$ and $\eta^a = 1 - (1 - p_a)\tau$.

It should be noticed that the problem of each member of dynasty a is the same as in the basic setting, where R is replaced by R^τ and X_t by X_t^τ. It follows that all results from Section 3

[25] In our setting an inheritance tax is equivalent to a tax on wealth as well as to a tax on capital income.

[26] The members of the egoistic dynasty do not cumulate any wealth and their effort decisions do not affect either the government budget constraint or total wealth in the economy. Only the proportion of types b matters, as it affects θ_t. We therefore can disregard their behavior.

and Appendix E apply to the model under the new normalization of variables. The effect of an inheritance tax τ can then be investigated using the characterization of the LRD as a function of the interest rate and the wage (see Figures $13-15$ in Appendix E). As it is shown in Appendix F, we obtain the following results, which are summarized in Table 4:

A - If the LRD without taxation is of type ZERO-WORK *or* FINI-WORK, *the tax has no effect on the type of LRD.*

B - If the LRD without taxation is of type MIX, *there exists a threshold $\tau^* \in (0,1)$ such that, when $\tau^* < \tau < 1$ the LRD with taxation is* FINI-WORK, *and when $\tau^* > \tau > 0$ the introduction of taxation could transform a* ZERO-MIX *LRD to a* FINI-MIX *LRD or vice versa.*

C - If the LRD without taxation is of type INFI-RENT, *there exist two thresholds $\tau^* \in (0,1)$ and $\tau^{**} \in (\tau^*, 1)$ such that the LRD becomes of type* MIX *(*ZERO-MIX *when $\xi > \ln(2-\beta)$ and $w < \mu_2\varepsilon$ and* FINI-MIX *otherwise) if $\tau^* < \tau < \tau^{**}$ and* FINI-WORK *if $\tau^{**} < \tau < 1$.*

<table>
<tr><td>$\tau = 0$</td><td colspan="4">Inheritance tax τ</td></tr>
<tr><td></td><td colspan="4">$0 < \tau \leq 1$</td></tr>
<tr><td>ZERO-WORK</td><td colspan="4">ZERO-WORK</td></tr>
<tr><td>FINI-WORK</td><td colspan="4">FINI-WORK</td></tr>
<tr><td></td><td colspan="2">$0 < \tau < \tau^*$</td><td colspan="2">$\tau^* < \tau \leq 1$</td></tr>
<tr><td>MIX</td><td colspan="2">MIX</td><td colspan="2">FINI-WORK</td></tr>
<tr><td></td><td>$0 < \tau < \tau^*$</td><td colspan="2">$\tau^* < \tau < \tau^{**}$</td><td>$\tau^{**} < \tau \leq 1$</td></tr>
<tr><td>INFI-RENT</td><td>INFI-RENT</td><td colspan="2">MIX</td><td>FINI-WORK</td></tr>
</table>

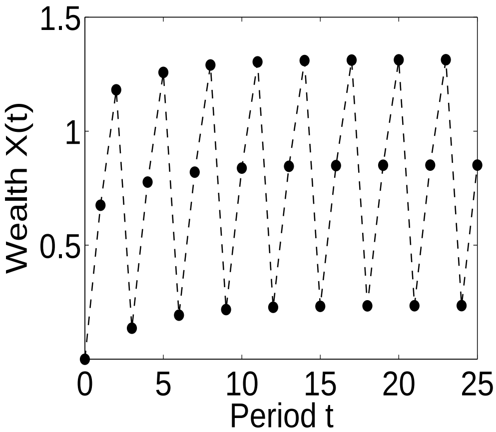
- Table 4 -

Some observations about the results are in order. First, although the introduction of an inheritance tax does not affect the type of LRD when this is initially of type ZERO-WORK or FINI-WORK, it affects the speed of convergence towards its long-run pattern and the level of wealth accumulated in the long-run, respectively. Consider the situation in which the LRD is ZERO-WORK and initial wealth is positive. As long as $X_t^\tau > 0$, transmitted wealth follows the following dynamic equation $X_{t+1}^\tau = \beta\eta^a R(X_t^\tau - \sigma)$. The smaller η^a, which is negatively related to τ, the faster the convergence of wealth towards zero. Similarly, the limiting value of transmitted wealth in a FINI-WORK LRD in the presence of taxation, $\widetilde{X}^\tau$, is increasing in R^τ, which in turn is decreasing in τ.

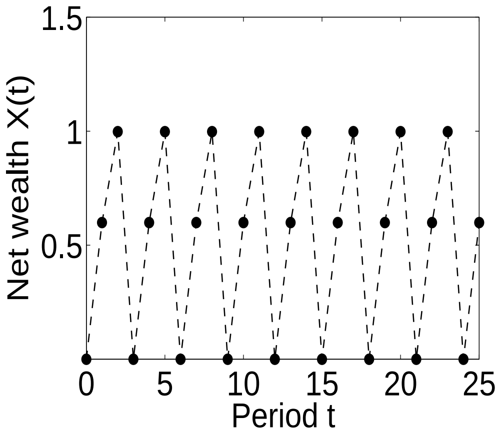
Second, points B and C indicate that there always exists a tax rate on inheritance high enough to force people to work. It follows that a confiscatory tax on inheritance can lead to the disappearance of *dilapidators* as well as of *ruiners* and/or *rentiers*. For instance, when $\tau > \tau^{**}$ and the LRD without taxation is of type MIX, we assist to the disappearance of *dilapidators* and also (when the initial LRD if ZERO-MIX) of *ruiners*. When $\tau > \tau^*$ and the LRD without taxation is INFI-RENT, the introduction of taxation makes the class of rentiers disappear.

Third, point B implies that starting from an INFI-RENT LRD the introduction of taxation always slows down the accumulation of wealth. While for low tax rates ($\tau < \tau^*$) the LRD remains of type INFI-WORK, its speed of convergence is slower, as $\beta\eta^a R$ is decreasing in τ. Moreover, by using medium tax rates ($\tau^* < \tau < \tau^{**}$) the government can always prevent a dynasty from cumulating an infinite amount of wealth, as the LRD becomes of type MIX. Furthermore, by using even higher tax rates ($\tau > \tau^{**}$) the government can implicitly force all the members of a dynasty to work and to transmit amounts of wealth that converge to the finite value $\widetilde{X}^{\tau}$.

Forth and most interesting, it is not always the case that the introduction of taxation reduces transmitted wealth both in the long-run and in each period. In fact, when the LRD without taxation is of type MIX the effect of taxation on wealth is ambiguous, as the tax can generate but also eliminate *ruiners.* The fact that inheritance tax eliminates *ruiners* is particularly paradoxical and could have interesting policy implications. It implies that taxing transmitted wealth could lead in certain periods to a higher wealth than in the context without taxation. The intuition for this result, which is also implied by point B, can be better understood by means of some numerical examples illustrated in Figures 3 – 7 when $\varepsilon = 1$, $\beta = 0.5$, $\xi = 0.7$, $R^0 = 1.5$ and $p_a = 0.25$. Consider first Figures 3 and 4, where, given $w = 1.9$, a tax of 15% makes the LRD go from a FIN-MIX to a ZERO-MIX.



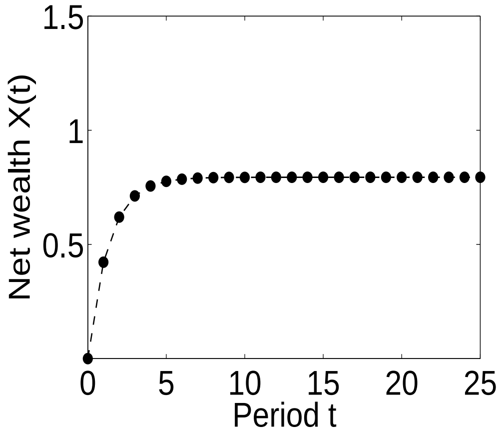
– Figure 3 –



– Figure 4 –

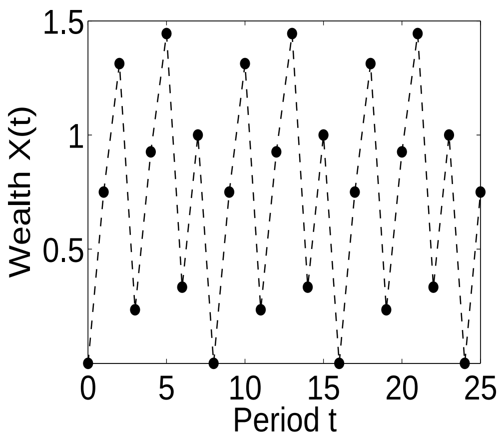
Without government intervention (Figure 3) we have (in the long-run) cycles composed of two generations who work and transmit positive wealth and one generation who does not work and squanders part of the wealth accumulated by her predecessors. The introduction of an inheritance tax leads the first two generations of the cycle to transmit a lower amount of wealth. This is high enough (greater than $\overline{X}$) to induce the third generation in the cycle not to work but insufficient to induce her to transmit a positive level of wealth. In other words, the third generation in the cycle from being a *dilapidator* becomes a *ruiner* (the resulting ZERO-MIX LRD is represented in Figure 4). In this example cumulated wealth is lower with taxation than in the scenario without taxation both in each period and in the long run. Although this result is standard in a classical setting it is not necessarily obvious in a context with endogenous effort choice.

We continue the numerical example by considering a higher tax rate, $\tau = 50\%$ (Figure 5). This high tax rate induces all generations to work, as the tax prevents any generation from leaving more than $\mathcal{X}$. The derived lifetime income in each period is high enough to induce each member to leave a positive wealth.

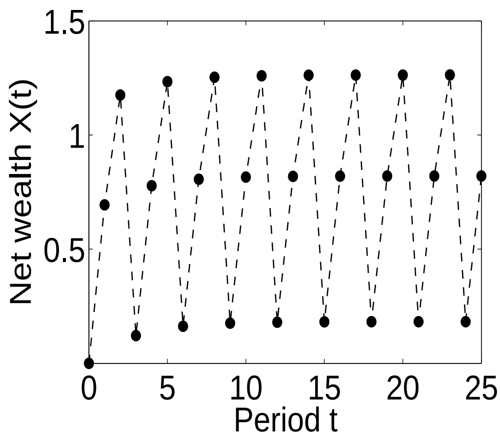


– Figure 5 –

When $\tau = 15\%$ a third of the generations transmits zero wealth, a third transmits 0.6 and the last third transmits 1. When $\tau = 50\%$ starting from the 5th generation all generations transmit around 0.8. This example therefore shows that by providing incentives to work, an inheritance tax can also provide incentives to transmit positive levels of wealth. In turn, the average long-run wealth with the high tax rate can be higher than the one corresponding to the milder tax rate.[27]



– Figure 6 –



– Figure 7 –

A second numerical example, in which we fix $w = 2$, shows that to make *ruiners* disappear taxation does not need to lead to a FINI-WORK LRD. In fact, the introduction of taxation can make

[27] Notice however that the welfare of the dynasty is lower with a higher tax rate because, even if the tax leads to a higher average wealth in the economy, the members of the dynasty are implicitly forced to work.

ruiners disappears also if it makes the LRD move from a ZERO-MIX to a FINI-MIX. It is equally possible that although with taxation the average wealth is lower, it is systematically higher for some generation. The following numerical example, illustrated in Figures 6 and 7, helps understanding this result. Without taxation (Figure 6) a *ruiner* appears each eight generations. The dynamic within each cycle is quite diverse. The first two generations work and cumulate wealth, which is partially dilapidated by the third generation. The following forth and fifth generations restart working and transmit increasing levels of wealth, which is afterwards partially dilapidated by the sixth generation and rebuilt by the seventh. The level of wealth transmitted by the seventh generation is high enough to induce the eight generation (*the ruiner)* not to work but not enough to induce her to leave positive wealth.

In this situation, an inheritance tax of 10% makes *ruiners* disappear. In fact the LRD becomes a FINI-MIX (Figure 7) where each two generations who work and cumulate wealth are followed by a third generation who does not work and dissipates only a portion of her inherited wealth. The reasons why the introduction of the tax has changed the dynamics of wealth in such a way hinges on the behavior of the seventh generation in the initial equilibrium.[28] Without taxation the seventh generation breaks the sequence of two workers and one dilapidator because she induces the eighth generation not only not to work but also not to leave any wealth. Conversely, with taxation the wealth accumulated by the seventh generation is lower (than without taxation) and does not provide enough incentives to induce the eighth generation not to work. Therefore, with taxation the eighth generation works and transmits a positive level of wealth. In other words, the inheritance tax by preventing the existence of *ruiners* maintains a link in the transmission of wealth across generations so that there is no generation who inherits zero wealth.

6. Extensions

In this section we discuss some possible extensions to our model. In particular we investigate the effect on the behavior of a dynasty of variable wage opportunities and of exogenous shocks.

The analysis of Section 2 holds for general price processes. However, the characterization of the long-run dynamics of Section 3 assumes that, as most theoretical models studying the dynamics of wealth accumulation and distribution, both wages and interest rates are constant over time. We study in the next two subsections (6.1 and 6.2) the robustness of our results to allowing wage opportunities to evolve over time.

Although we explicitly chose to abstract from luck we do recognize that luck is an important

[28]The beginning of the cycles in the two equilibria are otherwise the same: generations 1, 2, 4, 5 and 7 cumulate wealth while generations 3 and 6 dilapidate it.

determinant of the process of wealth accumulation. In fact there are at least two situations where we think that luck is crucial: the appearance of "self made (wo)men" and the "reverse of fortunes". We study in the last subsection (6.3) the effect of different types of shocks to economic variables on the dynamics generated by our model.

6.1 - Exogenous growth of wage opportunity.

In this subsection we partially relax the assumption of fixed prices and assume, coherent with Kaldorian facts, that wages grow at a constant positive rate γ, i.e., $w_t = (1+\gamma)^t w_0$. Under this assumption both the thresholds $\mathcal{X}_t$ and $\widetilde{X}$ increase over time and grow towards infinity. We establish in Appendix G that, there is always a period T' after which the wealth accumulated by a sequence of workers is greater than both $\widehat{X}$ and $\mathcal{X}_{T'+t}$.[29] Moreover, since the wage w_t eventually grows towards infinity, in all types of LRD wealth must tend towards infinity as well. Consequently, for each $w_0 > 0$, $\gamma > 0$ and $X_0 \geq 0$, there are only two possible LRD: when $\beta R > 1$ the economy grows unboundedly towards an INFI-RENT LRD, and when $\beta R < 1$ the economy converges towards a INFI-MIX LRD. The convergence towards the infinite wealth dynamics is monotonic in the first case but not in the second.

	$\beta R < 1$	$\beta R > 1$
For all $w_0 > 0$, $\gamma > 0$ and $X_0 \geq 0$	INFI-MIX	INFI-RENT

- Table 5 -

These results, summarized in Table 5, are quite intuitive. However, it should be noticed that they concern the long run only. They do not necessarily characterize the non-monotonic pattern that wealth can potentially follow in a shorter time span. For example, even in the case of an INFI-RENT LRD we can find along the way both *ruiners* and *dilapidators*. More in general, the medium term dynamics that can emerge in the context of exogenously growing wages can be compatible with a variety of dynastic behaviors observed in the real world.

6.2 - Unequal wage opportunity.

Our setting could also be used to look at the dynastic transmission of wealth in an unequal opportunity society, wherein the wage available to a generation depends on her inherited wealth. This corresponds to the realistic case where offsprings of wealthy families have access to higher paid jobs (see Stokey 1998 or Phelan 2006).[30] To model an "unequal opportunity" society we assume that there exist two increasing sequences $(w^0, ..., w^j, w^{j+1}, ...)$ and $(X^0, X^1, ..., X^j, X^{j+1}, ...)$. The

[29] Notice that the fact that wages grow over time does not affect the dynamic of wealth accumulation of a sequence of agents who do not work, $X_{t+1} = \beta R[X_t - \sigma\varepsilon]$. Therefore $\widehat{X}$ is independent of w.

[30] Offsprings of wealthy families may have access to better paid jobs if there exist unequal opportunities in the access to education or discrimination on the job market due to some type of social norm.

wage w_t proposed at time t to an agent who inherited the wealth X_t is $w^i \in (w^0, ..., w^j, w^{j+1}, ...)$, where i is the highest integer such that $X^i \leq X_t$, $X^i \in (X^0, X^1, ..., X^j, X^{j+1}, ...)$.[31]

In order to analyze the LRD in this context it is necessary to be more specific about the relation between the thresholds X^i that allow having access to better opportunities and the corresponding wage w^i. We study a basic case where initial wealth X_0 as well as the initial threshold X^0 are zero and where the other thresholds X^i, $i > 1$, correspond to $\mathcal{X}(w^{i-1})$, the minimum level of wealth that induces an agent with wage w^{i-1} not to work. Hence:

$$w_t = \begin{cases} w^0 & \text{if} \quad X_t < X^1 = \mathcal{X}(w^0) \\ & ... \\ w^j & \text{if} \quad X^j = \mathcal{X}(w^{j-1}) \leq X_t < X^{j+1} = \mathcal{X}(w^j) \\ & ... \end{cases}$$

When the wage of the first generation is sufficiently low, the LRD with unequal opportunity is the same as in our basic model. In fact, because $X_0 = 0$, when $w_0 < \sigma\varepsilon$ the first generation has no choice but to work and, due to her low lifetime income, she does not transmit wealth to the next generation, who then cannot benefit from better wage opportunities. The economy therefore converges to a ZERO-WORK LRD. A similar logic works for the FINI-WORK LRD, which is possible only when $\beta R < 1$ and $w_0 \in (\sigma\varepsilon, w_a)$, where w_a is such that $\widetilde{X}(w_a) = \mathcal{X}(w_a)$.[32]

It is when w_0 is sufficiently high (i.e. $w_0 > w_a$ if $\beta R < 1$, $w_0 > \sigma\varepsilon$ if $\beta R > 1$) that the hypothesis of unequal opportunity modifies the possible LRD. We distinguish two cases depending on whether the maximum level that the wage can reach over time w^{sup} is finite or infinite.[33] Our results are summarized in Table 6. The first column ($w_0 = w^{sup}$) corresponds to the results of the basic model with fixed wage (see Table 2) when $X_0 = 0$. The second and the third columns correspond

[31]Our hypothesis could also be refined by introducing an additional condition about the number of previous generations that must have transmitted a wealth greater than X^i in order for the next generation to benefit from wage w^i. This could account for a situation where the children of the "nouveaux riches", whose parents only recently became part of the most influential social network, have worse wage opportunities than children whose ancestors have belonged to such a network for many generations.

[32]For convention we assume that $w_a = \sigma\varepsilon$ when it does not exist a $w_a > \sigma\varepsilon$ such that $\widetilde{X}(w_a) = \mathcal{X}(w_a)$. According to Appendix G a FINI-WORK LRD exists if and only iff $\sigma\varepsilon < w_0 < w_a$. In this case, accumulated wealth $\widetilde{X}(w_0)$ is less than (the first unequal opportunity threshold) $X^1 = \mathcal{X}(w^0)$. Therefore there is no change in the actual wage opportunities of any generation and the economy converges towards the same FINI-WORK LRD as the one obtained when the wage is exogenous and equal in each period to $w = w_0 \in (\sigma\varepsilon, w_a)$.

[33]The first case corresponds to a situation where the number N of thresholds w^i (and the corresponding $N+1$ X^i) is finite or N is infinite but $\lim\limits_{N\to+\infty} w^N = w^{sup}$ ($\lim\limits_{N\to+\infty} X^N$) is finite. The first case emerges when there exists an infinite number of thresholds w^i and $\lim\limits_{N\to+\infty} w^N = w^{sup} = +\infty$. Assuming that the wage is bounded implies that there is a wealth threshold X^{sup} above which everybody is offered the wage w^{sup}.

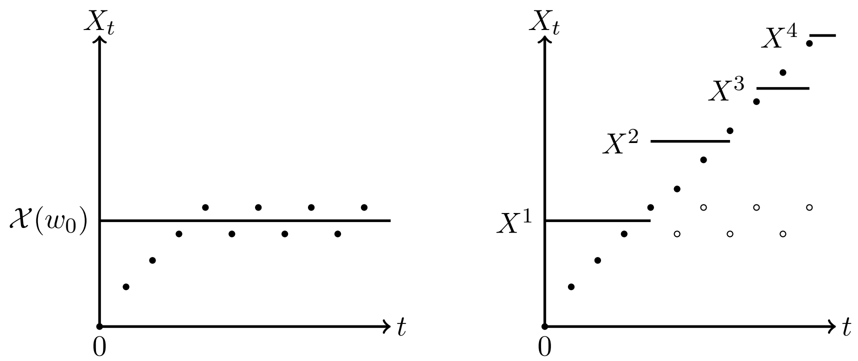
to the cases in which with unequal opportunity the maximum wage is bounded and unbounded, respectively.[34]

	Basic Model: $w_0 = w^{sup}$	$w_0 < w^{sup} < +\infty$	$w^{sup} = +\infty$
	When $\beta R < 1$ and $X_0 = 0$		
$w_0 < \sigma\varepsilon$	ZERO-WORK	ZERO-WORK	ZERO-WORK
$\sigma\varepsilon < w_0 < w_a$	FINI-WORK	FINI-WORK	FINI-WORK
$w_0 > w_a$	ZERO-MIX/FIN-MIX	FINI-MIX	INFI-WORK
	When $\beta R > 1$ and $X_0 = 0$		
$w_0 < \sigma\varepsilon$	ZERO-WORK	ZERO-WORK	ZERO-WORK
$w_0 > \sigma\varepsilon$ and $w^{sup} < w_b$	FINI-MIX ZERO-MIX INFI-RENT	FINI-MIX	INFI-WORK
$w_0 > \sigma\varepsilon$ and $w^{sup} > w_b$	FINI-MIX ZERO-MIX INFI-RENT	INFI-RENT	INFI-WORK

- Table 6 -

The key element to establish our results consists of the fact that if a member of the dynasty transmits a level of wealth greater than the first threshold X^1, the wealth accumulated by the future generations will pass all the other thresholds as well.

Interestingly when $w^{sup} = +\infty$, independent of βR we are in a situation as the one represented in Figure 8, where the introduction of unequal opportunities transforms a LRD of type FINI-MIX into one of type INFI-WORK.



– Figure 8 –

Starting from $X_0 = 0$ and initial constant wage w_0, the first four generations choose to work and accumulate wealth X_4. Without the unequal opportunity assumption, the wage remains constant

[34]When $\beta R > 1$, there exists a unique threshold wage $w_b < \widehat{X}$ such that: if w^{sup} is greater than w_b there exists a worker who receives less than X^{sup} but transmits more than both $\widehat{X}$ and X^{sup}.

and we assist to an infinite alternation of one *dilapidator* and of one generation who works and rebuilds the wealth dissipated by her predecessor. This FINI-MIX LRD is represented on the left part of Figure 8.

Under a situation of unequal opportunities, the wealth X_4 is such that the member of the fifth generation is offered a wage w^1 greater that w_0 (the wage received by his predecessor). This higher wage induces her as well as the following two generations to work. The corresponding wealth X_7 accumulated by the seventh generation becomes big enough to offer better wage perspectives to the eighth generation who is now offered $w^2 > w^1$. This generation and the following will therefore also be induced to work, and so on. We are in the case of an INFI-WORK LRD represented on the right of Figure 8.

Few remarks are in order about the implications of the different hypothesis on the wage process. First, among the possible scenarios considered so far, an INFI-WORK LRD is possible only in the unequal opportunity setting. Second, starting from a MIX or INFI-RENT LRD with constant wages, it is impossible for the economy to move to a FINI-WORK LRD as a consequence of a permanent increase in wage (see Section 4.2). The switch between these two types of LRD is also not possible when wages grow at an exogenous constant rate (see Section 6.1). The hypothesis of unequal opportunity is the only one that can explain the disappearance of *dilapidators* and/or *rentiers* implied by such a switch as a consequence of (potentially) increasing wages. Third and most important, the existence of unequal opportunities makes the change in the class structure analyzed in Section 4.1 an endogenous phenomenon.

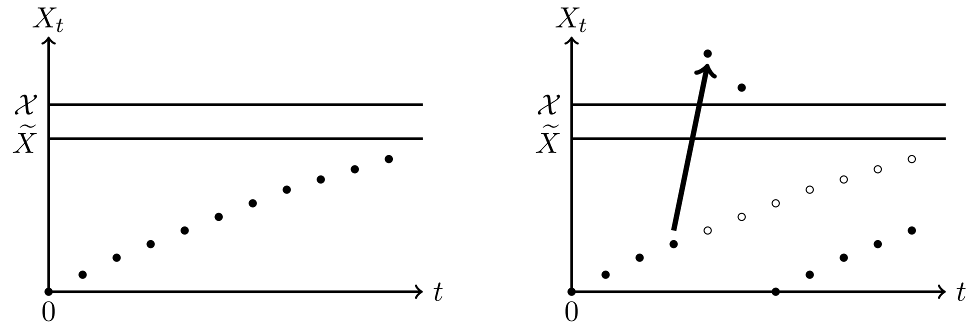
6.3. *Exogenous shocks.*

An easy way to consider the effect of luck in our model is the introduction of exogenous shocks. The shocks in our economy can affect the four fundamental exogenous parameters of the model (w, R, ξ, β and ε) or directly inherited wealth X_T at a certain period T. The effects of such shocks can be directly derived from the results of Section 3 and Appendix E and are briefly discussed in this subsection.

Let $V = \{w, R, \xi, \beta, \varepsilon\}$ be the initial values of the fundamental parameters. The corresponding LRD, given initial wealth X_0, follows Propositions 4 and 5.

If at a certain period T a permanent shock occurs to any of the fundamental parameters (that is, at time T the set of fundamental parameters becomes $V' \neq V$) the LRD becomes the one characterized in Propositions 4 and 5 given V' and initial wealth X_T (accumulated by the first T generations under V). The results of Appendix E allow determining precisely the impact of a change in one of the parameter on the dynamics of wealth accumulation.

We now turn to the effect of a transitory shock to the fundamental parameters. This situation corresponds to assuming that at a certain time T the set V changes to V' to return to its initial value at time $T' > T$. It should be noticed that the LRD obtained after such a temporary shock is equivalent to the one obtained in correspondence of the initial parameters V and a new (initial) wealth $X_{T'}$. It follows that, in order to understand the impact of transitory shocks it is enough to study the effect of an unexpected change in inherited wealth.



– Figure 9 –

The analysis of a big positive shock to wealth allows analyzing, among others, the consequences of the emergence of self-made men, like, for example, Bill Gate.

Figure 9 illustrates that even when a positive shock to wealth does not affect the type of LRD, it can have interesting effects on wealth accumulation in the short and medium run.

The left-hand side of Figure 9 shows a situation where, without the positive shock, the economy would converge monotonically to a FINI-WORK LRD. The right-hand side of Figure 9 depicts the situation where the member of the fourth generation makes, thanks to the exogenous positive shock, such a big fortune that $X_4 > \mathcal{X}$. As a consequence, the fifth generation chooses not to work and dilapidates part of inherited wealth. The next generation also chooses not to work and ruins all the wealth (i.e., $X_6 = 0$). From then on, all the following generations work and accumulate wealth, meaning that the long-run dynamics have not been affected by the temporary shock. However, paradoxically, the accumulation of wealth has been slowed down by the positive shock to wealth. In fact starting from the sixth period, an agent is richer when he belongs to the (otherwise equal) dynasty that hasn't received the shock than when he belongs to the dynasty that did receive the shock.

This example illustrates that the presence of positive shocks is another way our model can generate predictions that are consistent with the famous proverb "Shirtsleeves to shirtsleeves in three generations" as well as with the more recent evidence that 6 out of 10 families loose their fortunes by the end of the second generation and 9 out of 10 by the end of the third. In fact the existence of a member of the dynasty (a self-made man) who builds an extremely big patrimony may not be enough to guarantee the prosperity of all future members of the dynasty. This is because the hunger for accumulation β and the effort cost ξ are the crucial determinants, rather than initial wealth, of the LRD.

	$\beta R < 1$	$\beta R > 1$		
Switches after a big positive wealth shock	ZERO-MIX $\Updownarrow$ FINI-MIX	ZERO-WORK	ZERO-MIX $\Downarrow$ INFI-RENT	FINI-MIX.

- Table 7 -

We now turn to the long-run effect of a big positive shock to wealth. Table 7 summarizes the possible switches between types of LRD conditional on the hunger for accumulation relative to the interest rate. When $\beta R < 1$, initial wealth can only lead to a switch between a FINI-MIX and a ZERO-MIX LRD.[35] Wealth still plays a very important role when $\beta R > 1$, where a sufficient increase in wealth always allows switching from a ZERO-WORK, or a FINI-MIX, or a ZERO-MIX LRD to an INFI-RENT LRD.[36]

7. Conclusions

We have proposed a microfounded model to study the accumulation and transmission of wealth within a dynasty. For its focus on the dynamics of wealth accumulation the paper should be considered as a contribution complementary to both studies that calibrate stochastic growth models and theoretical models with human capital accumulation or imperfect credit markets.

We have shown that despite being deterministic the model can generate a variety of long-run patterns of wealth and effort choice. For instance, it can explain why some dynasties are captured into a poverty trap, why some other dynasties present *dilapidators* and *ruiners* who give rise to patterns of wealth as the one celebrated in the adage attributed to Andrew Carnegie, and why some dynasties consist of *rentiers,* who cumulate patrimonies that are meant to last indefinitely.

The proposed model can also explain some macroeconomic features of social mobility and class structure as well as the existence and the demise of the rich bourgeoisie. In particular, we show that it provides a deterministic explanation for the endogenous emergence of a stratified society as well as a possible interpretation for the demise of the rich bourgeoisie and the end of a class struggle, which does not need to appeal to capital markets imperfections.

The analysis of inheritance taxation in the context of our model contributes to the ongoing

[35]When $\beta R < 1$ the LRD is independent of initial wealth except when we have a dynamic of type MIX.

[36]When $\beta R > 1$, an INFI-RENT LRD can emerge once the positive shock to wealth is such that $X_{T'} > \widehat{X}$.

debate about the beneficial/detrimental effects of this type of taxation. Our study reveals that relatively high tax rates always induce people to work and as a consequence, paradoxically, may lead to higher long-run average wealth than mild tax-rates or no tax. In addition, inheritance taxation may redistribute wealth and lifetime income intergenerationally.

Lastly, we relax some of the assumptions used in the basic setting by investigating the effect on the LRD of two types of variable wage opportunities and of exogenous shocks. We have shown that these extensions give rise to new types of LRD but eliminate others. For instance, in presence of exogenous wage growth it is possible to find dynasties whose wealth tends towards infinity even when *dilapidators* appear periodically. Alternatively, the presence of "unequal opportunities" may foster the accumulation of wealth and provide incentives to work for dynasties that would otherwise be captured into a poverty trap or be stuck with the presence of members who squander part or the totality of wealth. Finally, the introduction of positive shocks to wealth allows analyzing the response of a dynasty to the existence of self-made men. In this respect, we find that a big positive shock to wealth may, paradoxically, slow down the long-run process of accumulation of the dynasty.

To summarize, our analysis reveals that the main factors determining the long-run dynamics of wealth and effort are the intensity of preferences towards wealth accumulation and the predisposition towards working or, alternatively, the entrepreneurial attitude. In most cases the interplay between these two preference parameters, rather than initial wealth or transitive shocks to wealth, determine the long-run process of wealth accumulation and transmission within a family lineage and the evolution of wealth inequalities.

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Appendix A – Proof of Proposition 1.

An agent chooses e_t and x_{t+1} in order to maximize (1) subject to the budget constraint $\Omega_t = w_t e_t + R_t x_t$. Therefore, given x_t, an agent maximizes: $\phi(x_{t+1}, e_t) = (1-\beta)\ln\left[w_t e_t + R_t x_t - x_{t+1}\right] + \beta \ln\left[\varepsilon + x_{t+1}\right] - \xi e_t$. It follows that the desired bequests $\breve{x}_{t+1}$ satisfies: $\phi'_x(\breve{x}_{t+1}, e_t) = -(1-\beta)/(\Omega_t - \breve{x}_{t+1}) + \beta/(\varepsilon + \breve{x}_{t+1}) = 0$. Hence: $\breve{x}_{t+1} = \beta\Omega_t - (1-\beta)\varepsilon$. Taking into account the non-negativity bequest constraint $x_{t+1} = \max\{\breve{x}_{t+1}, 0\}$ we obtain (2). □

Appendix B – Proof of Proposition 2.

According to Proposition 1, we have:

$$x_{t+1} = \begin{cases} 0 & \text{if} \quad (X_t \leq \sigma\varepsilon \ \text{ and } \ e_t = 0) \ \text{ or } \ (w_t + X_t \leq \sigma\varepsilon \ \text{ and } \ e_t = 1) \\ \beta[w_t + X_t - \sigma\varepsilon] \quad \text{if} \quad (w_t + X_t > \sigma\varepsilon \ \text{ and } \ e_t = 1) \\ \beta[X_t - \sigma\varepsilon] \quad \text{if} \quad (X_t > \sigma\varepsilon \ \text{ and } \ e_t = 0) \end{cases}.$$

When $X_t < \sigma\varepsilon - w_t$, the end-of-period wealth x_{t+1} is zero independent of the effort choice. In order to choose effort then the agent compares the utility he derives when he does not work $\phi(0,0)$ with the utility he derives when he works $\phi(0,1)$. Since $\phi(0,0) = (1-\beta)\ln X_t + \beta \ln \varepsilon$ and $\phi(0,1) = (1-\beta)\ln(w_t + X_t) + \beta\ln\varepsilon - \xi$, we have that $\phi(0,0) > \phi(0,1)$ if and only if $\xi > (1-\beta)\ln(1 + w_t/X_t)$. Then, when $X_t < \sigma\varepsilon - w_t$, $e_t = 0$ if and only if $X_t > w_t/(\tilde{e} - 1)$, where $\tilde{e} = e^{\xi/(1-\beta)}$. Taking into consideration that $w_t/(\tilde{e}-1) \leq \sigma\varepsilon - w_t$ if and only if $w_t \leq \mu_1\varepsilon$, where $\mu_1 = \sigma(1 - 1/\tilde{e})$, it follows that when $X_t < \sigma\varepsilon - w_t$, $e_t = 0$ if and only if:

$$X_t > X_t^\sharp = \begin{cases} w_t/(\tilde{e} - 1) & \text{if } w_t < \mu_1\varepsilon \\ \sigma\varepsilon - w_t & \text{if } w_t \geq \mu_1\varepsilon \end{cases}.$$

When $X_t > \sigma\varepsilon$, the end-of-period wealth x_{t+1} is positive independent of the effort choice. Similarly to the above case, in order to choose effort the agent compares the utility he derives when he does not work $\phi(x_+, 0)$ with the one he derives when he works $\phi(x_+, 1)$. Since $\phi(x_+, 0) = (1-\beta)\ln[(1-\beta)(X_t + \varepsilon)] + \beta\ln[\beta(X_t + \varepsilon)]$ and $\phi(x_+, 1) = (1-\beta)\ln[(1-\beta)(w_t + X_t + \varepsilon)] + \beta\ln[\beta(w_t + X_t + \varepsilon)] - \xi$, we have that $\phi(x_+, 0) > \phi(x_+, 1)$ if and only if $\xi > \ln[1 + w_t/(X_t + \varepsilon)]$. Then, when $X_t > \sigma\varepsilon$, $e_t = 0$ if and only if $X_t > w_t/(e^\xi - 1) - \varepsilon$. Since $w_t/(e^\xi - 1) - \varepsilon > \sigma\varepsilon$ if and only if $w > \mu_2\varepsilon$, where $\mu_2 = (e^\xi - 1)/\beta$, it follows that when $X_t > \sigma\varepsilon$, $e_t = 0$ if and only if:

$$X_t > \overline{X}_t = \begin{cases} \sigma\varepsilon & \text{if } w_t \leq \mu_2\varepsilon \\ w_t/(e^\xi - 1) - \varepsilon & \text{if } w_t > \mu_2\varepsilon \end{cases}.$$

When $\sigma\varepsilon - w_t < X_t \leq \sigma\varepsilon$, the agent chooses between working and leaving a positive bequest and neither working nor leaving any bequest. His optimal choice then depends on the comparison between $\phi(x_+, 1)$ and $\phi(0,0)$. We have that $\phi(0,0) > \phi(x_+, 1)$ if and only if $\xi > (1-\beta)\ln[(1-$

$\beta)(w_t + X_t + \varepsilon)/X_t] + \beta \ln[\beta(w_t + X_t + \varepsilon)/\varepsilon]$. Then, when $\sigma\varepsilon - w_t < X_t \leq \sigma\varepsilon$, $e_t = 0$ if and only if $\mathcal{A} = \varepsilon^{\beta} e^{\xi}/[\beta^{\beta}(1-\beta)^{1-\beta}] > \varphi(X_t) = (w_t + X_t + \varepsilon)/X_t^{1-\beta}$.

Since $\varphi'(X_t)$ has the same sign as $X_t - \sigma(w_t + \varepsilon)$, $\varphi(.)$ is decreasing on the interval $(\sigma\varepsilon - w_t, \sigma\varepsilon]$. Therefore, on this same interval, $\varphi(.)$ reaches its maximum at $\varphi(\sigma\varepsilon - w_t) = \varepsilon/[\beta(\sigma\varepsilon - w_t)^{1-\beta}]$ and its minumum at $\varphi(\sigma\varepsilon) = (\beta w_t + \varepsilon)/[\beta^{\beta}[(1-\beta)\varepsilon]^{1-\beta}]$. It follows that $\mathcal{A} > \varphi(\sigma\varepsilon - w_t)$ if and only if $w_t < \mu_1\varepsilon$ and $\mathcal{A} < \varphi(\sigma\varepsilon)$ if and only if $w_t > \mu_2\varepsilon$. Consequently, when $\sigma\varepsilon - w_t < X_t \leq \sigma\varepsilon$, $e_t = 0$ if and only if:

$$X_t > X_t^{\star} = \begin{cases} \sigma\varepsilon - w_t & \text{if } \; w_t \leq \mu_1\varepsilon \\ \text{Root of } [\mathcal{A} - \varphi(X_t)] & \text{if } \mu_1\varepsilon < w_t < \mu_2\varepsilon \\ \sigma\varepsilon & \text{if } \; w_t \geq \mu_2\varepsilon \end{cases} .$$

According to the previous thresholds, $e_t = 0$ if and only if: $X_t^{\sharp} < X_t \leq \sigma\varepsilon - w_t$, $\sigma\varepsilon - w_t \leq X_t^{\star} \leq X_t < \sigma\varepsilon$, and $\sigma\varepsilon \leq \overline{X}_t < X_t$. When $w < \mu_1\varepsilon$ we have that $X_t^{\sharp} < \sigma\varepsilon - w_t$, $X_t^{\star} = \sigma\varepsilon - w$, and $\overline{X}_t = \sigma\varepsilon$. Therefore, $e_t = 0$ if and only if $X_t > X_t^{\sharp}$. When $\mu_1\varepsilon < w < \mu_2\varepsilon$ we have that $X_t^{\sharp} = \sigma\varepsilon - w_t$, $\sigma\varepsilon - w < X_t^{\star} < \sigma\varepsilon$ and $\overline{X}_t = \sigma\varepsilon$. Then, $e_t = 0$ if and only if $X_t > X_t^{\star}$. When $w > \mu_2\varepsilon$ we have that $X_t^{\sharp} = \sigma\varepsilon - w_t$, $X_t^{\star} = \sigma\varepsilon$ and $\overline{X}_t > \sigma\varepsilon$. Then, $e_t = 0$ if and only if $X_t > \overline{X}_t$. It follows that $e_t = 0$ if and only if X_t is larger than the threshold $\mathcal{X}_t$ defined by:

$$\mathcal{X}_t = \begin{cases} X_t^{\sharp} = w_t/(e^{\xi/(1-\beta)} - 1) & \text{if } \; w_t \leq \mu_1\varepsilon \\ X_t^{\star} = \text{Root of } \left\{\varepsilon^{\beta} e^{\xi}/[\beta^{\beta}(1-\beta)^{1-\beta}] - (w_t + X_t + \varepsilon)/X_t^{1-\beta}\right\} & \text{if } \; \mu_1\varepsilon < w_t < \mu_2\varepsilon \\ \overline{X}_t = w_t/(e^{\xi} - 1) - \varepsilon & \text{if } \; w_t \geq \mu_2\varepsilon \end{cases} ,$$

where it should be noticed that $X_t^{\sharp} \leq \sigma\varepsilon - w_t < X_t^{\star} < \sigma\varepsilon \leq \overline{X}_t$. □

Appendix C – Characterization of the wealth dynamics as a function of the wage. According to Propositions 1 and 2 we can distinguish the five following dynamics of wealth accumulation:

(a) If $w < \mu_1\varepsilon$ then $X_{t+1} = \begin{cases} 0 \quad \text{if} \quad X_t \leq \sigma\varepsilon \\ \beta R\Big[X_t - \sigma\varepsilon\Big] \quad \text{if} \quad X_t > \sigma\varepsilon \end{cases}$

(b) If $\mu_1\varepsilon < w < \min(\sigma\varepsilon, \mu_2\varepsilon)$ then $X_{t+1} = \begin{cases} 0 \quad \text{if} \quad X_t \leq \sigma\varepsilon - w \text{ or } X^{\star} < X_t \leq \sigma\varepsilon \\ \beta R\Big[w + X_t - \sigma\varepsilon\Big] \quad \text{if} \quad \sigma\varepsilon - w \leq X_t \leq X^{\star} \\ \beta R\Big[X_t - \sigma\varepsilon\Big] \quad \text{if} \quad X_t > \sigma\varepsilon \end{cases}$

(c) If $\mu_2\varepsilon < w < \sigma\varepsilon$ then $X_{t+1} = \begin{cases} 0 \quad \text{if} \quad X_t \leq \sigma\varepsilon - w \\ \beta R\Big[w + X_t - \sigma\varepsilon\Big] \quad \text{if} \quad \sigma\varepsilon - w \leq X_t \leq \overline{X} \\ \beta R\Big[X_t - \sigma\varepsilon\Big] \quad \text{if} \quad X_t > \overline{X} \end{cases}$

(d) If $\sigma\varepsilon < w < \mu_2\varepsilon$ then $X_{t+1} = \begin{cases} 0 \quad \text{if} \quad X^\star < X_t \leq \sigma\varepsilon \\ \beta R\Big[w + X_t - \sigma\varepsilon\Big] \quad \text{if} \quad 0 \leq X_t \leq X^\star \\ \beta R\Big[X_t - \sigma\varepsilon\Big] \quad \text{if} \quad X_t > \sigma\varepsilon \end{cases}$

(e) If $w > \max(\sigma\varepsilon, \mu_2\varepsilon)$ then $X_{t+1} = \begin{cases} \beta R\Big[w + X_t - \sigma\varepsilon\Big] \quad \text{if} \quad 0 \leq X_t \leq \overline{X} \\ \beta R\Big[X_t - \sigma\varepsilon\Big] \quad \text{if} \quad X_t > \overline{X} \end{cases}$ $\square$

Appendix D – Proof of Propositions 4 and 5.

In order to characterize the types of LRD of our economy, we establish some property of the non trivial branches of the dynamic equation (3). In particular, after some easy but tedious calculations, we characterize in the following Lemma, the form of the k-th element, monotonicity, and convergence for each of the two branches. Let $\widetilde{X} = \beta R[w - \sigma\varepsilon]/(1 - \beta R)$ and $\widehat{X} = \beta R\sigma\varepsilon/(\beta R - 1)$:

Lemma 1

A – Let X_T ,..., X_{T+k} be a sequence such that for $t \in \{0, k\}$, $X_{T+t+1} = \beta R\Big[w + X_{T+t} - \sigma\varepsilon\Big]$.

a) For all $t \in \{0, k\}$, $X_{T+t} \equiv \Phi_{X_T}(t) = (\beta R)^t[X_T - \widetilde{X}] + \widetilde{X}$.

b) $\Phi_{X_T}(t+1) - \Phi_{X_T}(t)$ has the sign of $(X_T - \widetilde{X})(\beta R - 1)$. Then, when $w \geq \sigma\varepsilon$, $\Phi_{X_T}(t+1) - \Phi_{X_T}(t)$ is positive if $\beta R > 1$ and has the sign of $\widetilde{X} - X_T$ if $\beta R < 1$. When $w < \sigma\varepsilon$, $\Phi_{X_T}(t+1) - \Phi_{X_T}(t)$ is negative if $\beta R < 1$ and has the sign of $X_T - \widetilde{X}$ if $\beta R > 1$.

c) $\lim\limits_{t\to+\infty} \Phi_{X_T}(t) = \widetilde{X}$ if $\beta R < 1$, $-\infty$ if ($\beta R > 1$ and $X_T < \widetilde{X}$) and $+\infty$ if ($\beta R > 1$ and $X_T > \widetilde{X}$).

B – Let X_T ,..., X_{T+k} be a sequence such that for $t \in \{0, k\}$, $X_{T+t+1} = \beta R\Big[X_{T+t} - \sigma\varepsilon\Big]$.

d) For all $t \in \{0, k\}$, $X_{T+t} \equiv \Psi_{X_T}(t) = (\beta R)^t(X_T - \widehat{X}) + \widehat{X}$.

e) $\Psi_{X_T}(t+1) - \Psi_{X_T}(t)$ is negative if $\beta R < 1$ and has the sign of $X_T - \widehat{X}$ if $\beta R > 1$.

f) $\lim\limits_{t\to+\infty} \Psi_{X_T}(t) = \widehat{X} < 0$ if $\beta R < 1$, $-\infty$ if ($\beta R > 1$ and $X_T < \widehat{X}$) and $+\infty$ if ($\beta R > 1$ and $X_T > \widehat{X}$).

One last element to introduce before characterizing the LRD of our economy is $\Delta_{X_0}(t)$, which denotes the complete trajectory of wealth, starting from X_0, when wealth follows:

$$X_{t+1} = \begin{cases} \beta R[X_t + w - \sigma\varepsilon] & \text{if} \quad 0 \leq X_t < \mathcal{X} \\ \beta R[X_t - \sigma\varepsilon] & \quad X_t > \mathcal{X} \end{cases}$$

• A necessary and sufficient condition (hereafter, *nsc*) to obtain a ZERO-WORK LRD is that: (i) $\forall t$ such that $X_t = 0$ we have $X_{t+1} = 0$ and (ii) there exists a period T such that $X_T = 0$. According to Appendix C, (i) is satisfied if and only if $w < \sigma\varepsilon$. Then, a *nsc* to obtain a ZERO-WORK LRD is that $w < \sigma\varepsilon$ and (ii). When $w < \sigma$, according to Appendix C and Lemma 1, (ii) is satisfied $\forall X_0$ if $\beta R < 1$.

• To obtain a FINI-WORK LRD it is necessary that $\lim_{t\to+\infty} \Phi_{X_T}(t) = \widetilde{X} > 0$, i.e., that $\beta R < 1$ and $\widetilde{X} > 0$. Using Lemma 1, this is equivalent to requiring $\beta R < 1$ and $w \geq \sigma\varepsilon$. Another necessary condition is that $\widetilde{X} \leq \mathcal{X}$. To see why this is the case, suppose the opposite was true, i.e. $\mathcal{X} < \widetilde{X}$. Since $\beta R < 1$, independent of X_0, there would exist a period in which wealth will be greater than $\mathcal{X}$. But then, the agents will choose to stop working, which (under $\beta R < 1$) would prevent wealth to converge towards $\widetilde{X}$. It follows that necessary conditions to obtain a FINI-WORK LRD are: $\beta R < 1$, $w \geq \sigma\varepsilon$, and $\widetilde{X} \leq \mathcal{X}$. According to dynamics (d) and (e) of Appendix C, these necessary conditions are also sufficient.

• A *nsc* to obtain an INFI-RENT LRD is that $\beta R > 1$ and there exists a T such that $X_T > \widehat{X}$. According to Lemma 1, these conditions are sufficient because they guarantee the existence of a $T' \geq T$ such that $\lim_{t\to+\infty} X_{T'+t} = +\infty$. They are also necessary. Indeed, if $\beta R < 1$ $\lim_{t\to+\infty} X_t$ is finite. If $\beta R < 1$ and $\nexists\, T$ such that $X_T > \widehat{X}$, then $\lim_{t\to+\infty} X_t = -\infty$.

• An obvious necessary condition to obtain a ZERO-MIX LRD is the existence of a T such that $X_T = 0$. It is also necessary that when $X_T = 0$: (i) $X_{T+1} > 0$ and (ii) there exists an m such that $X_{T+m} = 0$. According to Appendix C, (i) implies $w \geq \sigma\varepsilon$ and (ii) implies $w < \mu_2\varepsilon$. It follows that to have a ZERO-MIX LRD, it is necessary to be in the case (d) of Appendix C. In such a setting, a *nsc* to have a ZERO-MIX LRD is that wealth eventually becomes zero both starting from $X_T = 0$ and from X_0. That is, there must exist t and t' such that $\Delta_{X_0}(t)$ and $\Delta_0(t') \in (X^\star, \sigma\varepsilon)$. We can find more specific conditions for the case with $\beta R < 1$. In fact, when $\widetilde{X} < X^\star$ it is impossible for a t such that $\Delta_0(t) \in (X^\star, \sigma\varepsilon)$ to exist. Conversely, according to Lemma 1, some t and t' such that $\Delta_{X_0}(t)$ and $\Delta_0(t') \in (X^\star, \sigma\varepsilon)$ always exist when $X^\star < \widetilde{X} < \sigma\varepsilon$.

• Since our model is deterministic, there exists a unique LRD for any given dynasty. If the long run accumulation is monotonic, wealth accumulated can only: be zero (and necessarily we have a ZERO-WORK LRD); converge to a finite value (and necessarily we have a FINI-WORK LRD); or grow towards infinite (and necessarily we have a INFI-RENT LRD). Among the LRD with non monotonic long-run accumulation, only two cases can arise, depending on whether accumulated wealth can be zero or not. The first one corresponds to a ZERO-MIX LRD whereas the second one corresponds to a FINI-MIX LRD. Then, a *nsc* to obtain a FINI-MIX LRD it that the necessary and sufficient conditions to obtain ZERO-WORK, FINI-WORK, INF-RENT and ZERO-MIX LRD are not satisfied. □

Appendix E – Prices and long run dynamics.

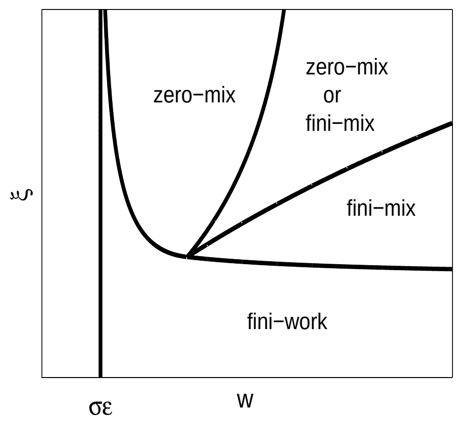
In this appendix we provide a graphical representation of the results of Propositions 4 and 5 in order to study the role of prices (w and R) and of the effort parameter ξ on the long run dynamics.[37]

[37] A derivation of the graphical illustrations is available from the authors upon request.

E.1 - Effort cost, wage opportunity and Long Run Dynamics.

We start with the analysis of the type of LRD obtained as a function of the effort cost ξ of the members of the dynasty and the wage w prevailing in the economy. Throughout this section we focus on the case where the wage w is greater than $\sigma\varepsilon$.[38] We distinguish three cases according to the value of βR.

Consider first the situation, depicted in Figure 10, where $\beta R < 1$. For low effort costs, independent of the wage, in the long run all generations work and transmit increasingly positive levels of wealth. Conversely for high effort costs, when the wage is low all generations are forced to work but, when the wage is high there are some generations who decide to stop working and to live with their inheritance. It is important to remark that what drives some of these (high wage) dynasties not to work is not a wealth effect. It is the fact that while the threshold $\mathcal{X}$ above which an agent decides not to work is decreasing in ξ, the limiting value of wealth $\widetilde{X}$ that can be accumulated by a dynasty of workers is independent of it. Therefore, while dynasties with low ξ work and can allow dynastic wealth to converge towards $\widetilde{X}$, dynasties with high ξ stop working before their wealth can approach $\widetilde{X}$ and (fully or partially) dilapidate their wealth.

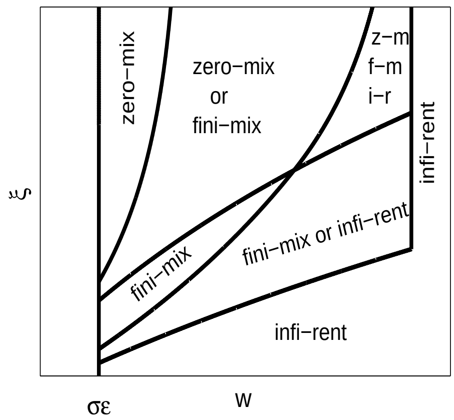


– Figure 10 –

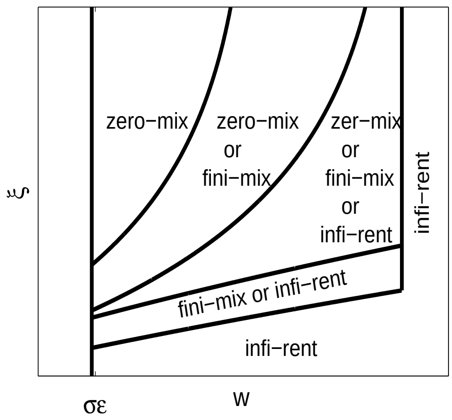
For given effort cost ξ, an increase in wage leads at the same time to an increase in the threshold $\mathcal{X}$ and in the limiting wealth $\widetilde{X}$. However, it can be shown that $\widetilde{X}$ increases faster than $\mathcal{X}$, making the existence of *dilapidators* and/or *ruiners* (i.e., the possibility that $\mathcal{X} < \widetilde{X}$) more plausible. Hence, an increase in wage allows going from a dynamics of type FINI-WORK to a dynamics of type MIX. In addition, among these MIX dynamics, there exist a (unique) wage below which the LRD is ZERO-MIX and above which it is FINI-MIX.

[38] This case is not restrictive. In fact when $\beta R < 1$ and $w < \sigma\varepsilon$, we have a ZERO-WORK LRD. This is also the case when $\beta R > 1$ and $(w < \mu_1\varepsilon)$ or $(X_0 = 0$ and $\mu_1 < w < \sigma\varepsilon)$. Considering only $w > \sigma\varepsilon$ avoids dealing with the possibility of a FINI-MIX and INFI-RENT LRD when $\mu_1\varepsilon < w < \sigma\varepsilon$. Cases that can emerge only for $\beta R > 1$ and some ranges of strictly positive X_0. Although the study of these cases is possible (and available from the authors upon request) it is extremely tedious and non-informative and is therefore omitted.

Consider now $\beta R > 1$. We illustrate in Figure 11 and 12 the case where $1 < \beta R < 2$ and $\beta R > 2$, respectively, where it is assumed that $X_0 < \widehat{X}$.[39] As the cost of effort increases there is a possibility (when the wage is not too high, depending on initial wealth X_0) to fall into a LRD with *ruiners*. Similarly, the set of effort costs in correspondence to which we have an INFI-RENT LRD gets wider as the wage increases. This is because the higher w the higher the wealth accumulated by a chain of workers. Therefore, the threshold $\widehat{X}$ (independent of w) that inherited wealth has to pass in order to have an INFI-RENT LRD is easier to reach. In fact, for high wages this is the only LRD, independent of ξ.



– Figure 11 –



– Figure 12 –

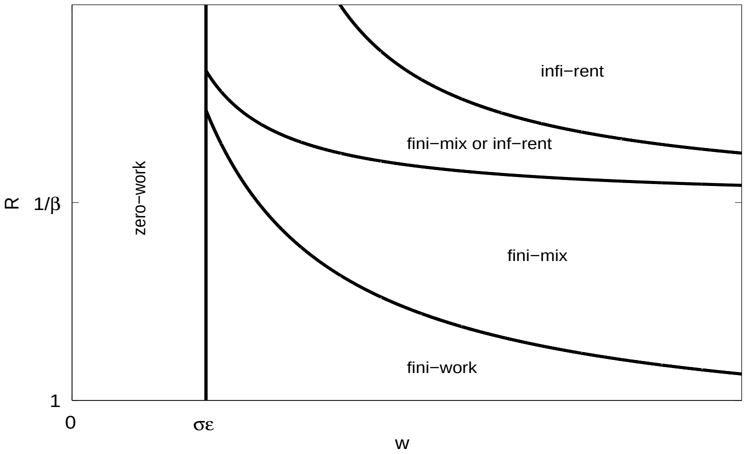
E.2 - Interest rate, wage opportunity and Long Run Dynamics.

We use Propositions 4 and 5 simultaneously to consider the LRD as a function of w and R. Since there are too many possible configurations, we focus only on the case where initial wealth X_0 is zero (or, by continuity, sufficiently low). We distinguish three cases according to the value of ξ (represented in Figure 13, 14, and 15, respectively): low effort cost, $\xi < \ln(2-\beta)$; intermediate effort cost, $\ln(2-\beta) < \xi < \ln[1/\beta]$; and high effort cost, $\xi > \ln[1/\beta]$.[40]

When the wage is relatively low (i.e., $w < \sigma\varepsilon$), independent of the effort cost ξ the interest rate R does not affect the type of LRD, which is of type ZERO-WORK. Consider now wages greater than $\sigma\varepsilon$. When the cost of effort is low (Figure 13), the higher the wage the higher the possibility of having agents who do not work. In particular there exist two thresholds for the interest rate, R_1 and R_2 ($0 < R_1 < R_2$), such that: if R is lower than R_1 the LRD is of type FINI-WORK; if R is greater than R_2 the LRD is of type INFI-RENT; and when R is in between R_1 and R_2 the LRD is of type FINI-MIX.

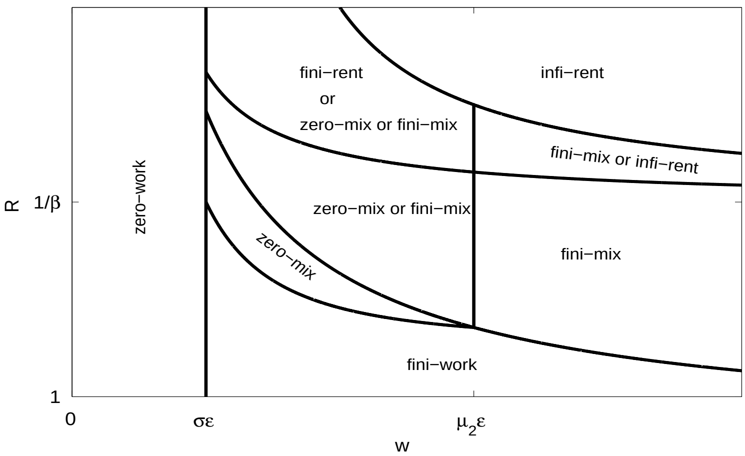
[39] When $\beta R > 1$ and $X_0 > \widehat{X}$ there is only an INFI-RENT LRD.

[40] Alternatively, in terms of the hunger for accumulation these three cases correspond to $\beta < 2-e^{\xi}$, $2-e^{\xi} < \beta < e^{-\xi}$, and $\beta > e^{-\xi}$, respectively.



– Figure 13 –

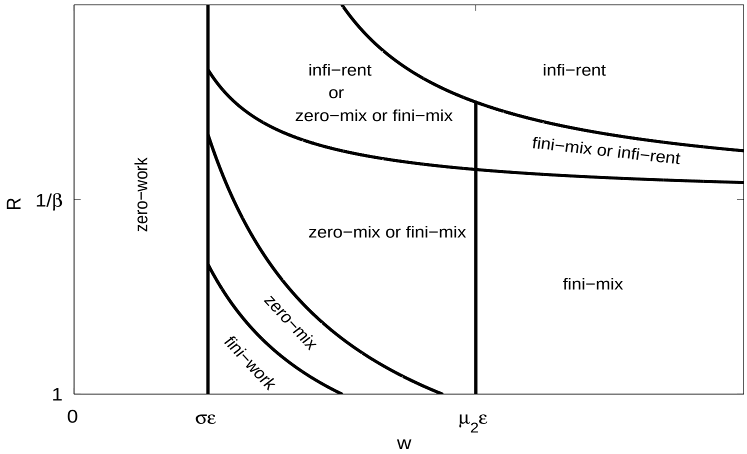
The possible switch from a FINI-WORK to a FINI-MIX LRD is due, different from subsection E.1, to a wealth effect. In fact, $\widetilde{X}$ is increasing in R but the threshold $\mathcal{X}$ is independent of it. Conversely, the possible switch from a MIX to an INFI-RENT LRD is not necessarily due to a wealth effect, as $\widehat{X}$ is decreasing in R. Intuitively, because the wealth of *rentiers* grows at a rate βR, the higher βR the lower the minimum initial wealth needed to obtain at a certain period a given level of wealth.



– Figure 14 –

The types of switches among LRD described above emerge also for intermediate effort costs (Figure 14) but only when the wage is sufficiently high, i.e. $w > \mu_2\varepsilon$. For intermediate wage levels, i.e. $\sigma\varepsilon < w < \mu_2\varepsilon$, changes in the interest rate can generate as well as eliminate *ruiners*. In fact when the wage is in this range, a direct switch from a FINI-WORK to a FINI-MIX LRD is no longer possible. As R increases the LRD must first move from FINI-WORK to ZERO-MIX. Only then it could go through a FINI-MIX and eventually become INFI-RENT.

Ruiners can emerge because the actual value of wealth received from the previous generation may not be high enough to convince an agent at the same time not to work and to leave a positive wealth. For a given bequest, as the interest rate reaches a certain threshold, the agent is induced to transmit a positive wealth so that the initial *ruiner* at higher interest rates becomes a *dilapidator*. For analogous reasons, as the interest rate increases further, the *dilapidator* becomes a *rentier*.

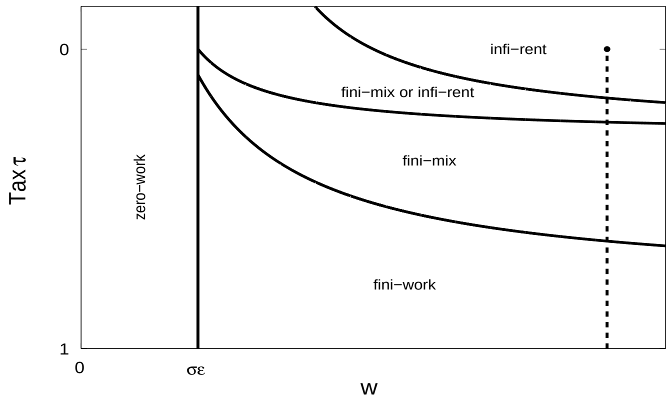


– Figure 15 –

The analysis of Figures 12 – 15 also point to the fact that the higher the effort cost the less plausible is to have a dynasty of all workers. In fact when the cost of effort is high (Figure 15) a FINI-WORK LRD exists only for a very small range (and low values) of w and R. In particular a FINI-WORK LRD exists only for wages strictly lower than $\mu_2\varepsilon$, implying that it is no longer possible to go directly from a FINI-WORK to a FINI-MIX LRD. In the remaining cases the effect of an increase in the interest rate is equivalent to the ones discussed above. □

Appendix F – Inheritance Taxation.

In order to determine the role of an inheritance taxe τ, it is sufficient to use the graphical representation of the LRD on the space (w, R). In fact, the case $\tau = 0$ corresponds to $R^\tau = R$ and, the case $\tau = 1$ is equivalent to $R^\tau = 0$. Starting from Figure 13, 14 or 15, for any given w et R, we first have to localize the situation (point) without taxation.



– Figure 16 –

Starting from this point, we can draw a vertical line representing the evolution of the type of LRD as we increase τ. Figure 16 represents the situation, corresponding to Figure 13, where without taxation the LRD is INFI-RENT (bolded point). By following the shaded line it is easy to see that there exist two taxe rates $\tau^* \in (0,1)$ and $\tau^{**} \in (\tau^*, 1)$ such that the LRD becomes FINI-MIX if $\tau^* < \tau < \tau^{**}$ and FINI-WORK if $\tau^{**} < \tau < 1$. The same methodology, that applies to any other initial point and to Figure 14 and 15, allows us to obtain the results summarized Table 4. □

Appendix G – Wage opportunities.

G.1 - Exogenous growth of wage opportunity.

When wages grow at fixed positive rate γ, i.e. $w_t = (1+\gamma)^t w_0$, the threshold $\mathcal{X}_t$ is no longer constant over time. In fact, when at a certain period T the wage becomes sufficiently high (i.e. $w_T = (1+\gamma)^T w_0 > \max\{\mu_2\varepsilon, \sigma\varepsilon\}$), le threshold $\mathcal{X}_t$ is represented by $\overline{X}_t$ and in period $T+t$ is therefore $\mathcal{X}_{T+t}(w_T) = (1+\gamma)^t w_T/(e^\xi - 1) - \varepsilon$. This threshold increases over time in a convex way and tends towards infinity. While the threshold $\mathcal{X}$ still exists, this is no longer the case for the finite threshold $\widetilde{X}$ that can be accumulated by an infinite sequence of workers when $\beta R < 1$.

In fact, starting from any given value of X_T and w_T, successive iterations of wealth according to the dynamic equation $X_{T+t+1} = \beta R[w_{T+t+1} + X_{T+t} - \sigma\varepsilon]$ lead to:

$$\begin{aligned} X_{T+t}(w_T) &= w_T[(1+\gamma)^{T+t}\beta R + (1+\gamma)^{T+t-1}(\beta R)^2 + ... + (1+\gamma)^{T+1}(\beta R)^t] \\ &\quad -\sigma\varepsilon[\beta R + (\beta R)^2 + ... + (\beta R)^t] + (\beta R)^t X_T(w_T) \end{aligned}$$

Independent of X_0, there always exists a date T' such that $X_{T'+t}$ converges towards $+\infty$ as t goes to $+\infty$. Therefore, independent of βR, the wealth accumulated by a infinite sequence of workers eventually goes to infini. Importantly, there always exists a T'' such that $cx - d > 0$, where $c = (1+\gamma)^{T''} w_0$, $x = \beta R$, and $d = w_0/(e^\xi - 1)$. By letting $a = X_T \geq 0$, $b = \sigma\varepsilon$ and $y = 1+\gamma$ it follows that:

$$\mathcal{J}_t = X_{T''+t} - \mathcal{X}_{T''+t}(w_{T''}) = ax^t - b(x + x^2 + ... + x^t) + y^t\left[cx - d + \frac{cx^2}{y} + \frac{cx^3}{y^2} + ... + \frac{cx^t}{y^{t-1}}\right] + \varepsilon,$$

and, since $y > 1$ and $cx > d$, $\lim_{t\to+\infty} \mathcal{J}_t = +\infty$. Consequently, even if the threshold of wealth above with an agent decide not to work increases and tends towards infinity, eventually the wealth accumulated by a sequence of workers is even greater, that is $X_{T''+t} > \mathcal{X}_{T''+t}$, and it is also grater than $\widehat{X}$.

As summarized in Table 5, when wages grow at a fixed positive rate, independent of X_0, there are only two possible cases: when $\beta R < 1$ the economy converges towards a INFI-MIX LRD and, when $\beta R > 1$ the economy converges towards a INFI-RENT LRD.

G.2 - Unequal wage opportunity.

The key element to establish the results (Table 6) of the analysis of unequal wage opportunity is that if the wealth of some member of the dynasty becomes greater than the first threshold X^1, cumulated wealth will evenutally pass all the other thresholds X^i, $i > 1$, as well. This property is straightforward when $\beta R > 1$, where wealth accumulated by a sequence of workers tends towards infinity. In order to show that it holds also when $\beta R < 1$ and $w \geq \sigma\varepsilon$ we need to compare $\widetilde{X}$ and $\mathcal{X}$, when $\mathcal{X} = X^\star$ or $\overline{X}$.

When $\mathcal{X} = X^\star$ ($w \in [\sigma\varepsilon, \mu_2\varepsilon]$) we are interested in the sign of $\Theta(w) = X^\star(w) - \widetilde{X}(w)$, where $\Theta'(w) = X^{\star\,\prime}(w) - \beta R/(1-\beta R)$ and $\Theta''(w) = X^{\star\prime\prime}(w)$. Using the same notation as in Appendix B, we have let $\varphi(X^\star, w) - \mathcal{A} = 0$ and, using the implicit function theorem we have that $X^{\star\,\prime}(w) = -\varphi_w/\varphi_X$. Since $\varphi_X = [\beta X^\star - (1-\beta)(w+\varepsilon)]/X^{\star 2-\beta}$ and $\varphi_w = X^{\star\beta-1}$ we have $X^{\star\,\prime}(w) = X^\star(w)/[(1-\beta)(w+\varepsilon) - \beta X^\star(w)]$. It follows that $X^{\star\,\prime}(w)$ has the sign of $\sigma(w+\varepsilon) - X^\star(w)$. Because $X^\star(w) \leq \sigma\varepsilon$, $X^{\star\,\prime}(w) > 0$. Moreover, $X^{\star\,\prime\prime}(w)$ has the sign of $(w+\varepsilon)X^{\star\,\prime}(w) - X^\star(w)$, that is, the sign of $w + \varepsilon + X^\star$. Hence, we have $\Theta''(w) > 0$. We also have $\Theta(\sigma\varepsilon) = X^\star(\sigma\varepsilon) > 0$ and $\Theta(\mu_2\varepsilon) = \sigma\varepsilon - \beta R[\mu_2 - \sigma]\varepsilon/(1-\beta R)$. Then, $\Theta(\mu_2\varepsilon)$ has the sign of $(1-\beta)/(\beta R) - e^\xi + 1$. Since $X^\star(\mu_2\varepsilon) = \sigma\varepsilon$, $\Theta'(\mu_2\varepsilon)$ has the sign of $(1-\beta R)/(\beta R) - e^\xi + 1$. Furthermore, since $(1-\beta R)/(\beta R) < (1-\beta)/(\beta R)$ we can conclude that: (i) when $\xi \geq \tilde{\xi} = \ln(1 + \sigma/R)$ we have $\Theta(\sigma\varepsilon) > 0$, $\Theta(\mu_2\varepsilon) < 0$, and $\Theta'(\mu_2\varepsilon) < 0$. It follows that $\Theta(.)$ is strictly decreasing and there exists a unique $w_a \in (\sigma\varepsilon, \mu_2\varepsilon)$ such that we have $\widetilde{X} < X^\star$ when $w < w_a$ and $\widetilde{X} > X^\star$ when $w > w_a$. (ii) When $\xi < \tilde{\xi}$ we have $\Theta(\sigma\varepsilon) > 0$, $\Theta(\mu_2\varepsilon) > 0$ and $\theta''(\mu_2\varepsilon) > 0$. Also, $\Theta(.)$ is either decreasing, or first decreasing and then increasing. When it is decreasing it must be $\widetilde{X} < X^\star$. Conversely, when $\Theta(.)$ is first decreasing and then increasing there exists a unique value w_{ro} such that $\Theta'(w_{ro}) = 0$. In this case, $X'^{\,\star}(w_{ro}) = \beta R/(1-\beta R)$ and, hence, $X^\star(w_{ro}) = \beta R(1-\beta)(w_{ro}+\varepsilon)/[1-\beta R(1-\beta)]$ and:

$$\Theta(w_{ro}) = \left[\frac{\beta R(1-\beta)}{1-\beta R(1-\beta)} - \frac{\beta R}{1-\beta R}\right] w_{ro} + \left[\frac{\beta R(1-\beta)}{1-\beta R(1-\beta)} - \frac{(1-\beta)R}{1-\beta R}\right]\varepsilon.$$

Simple calculations reveal that $\Theta(w_{ro})$ has the sign of $\beth \equiv -w_{ro} + \sigma\varepsilon[1/\beta - (R-1)]$. The term $\beth$ reaches its minumum when w_{ro} takes its maximum value, that is $w_{ro} = \sigma\varepsilon/(\beta R)$. Hence, $\beth \geq \sigma\varepsilon(R-1)[1/(\beta R) - 1] \geq 0$ and $\Theta(w_{ro}) > 0$. Because $\Theta(w_{ro})$ is the minimum of $\Theta(.)$, it follows that when $\Theta(.)$ is decreasing and then increasing $\Theta(w) > 0$, $\forall w \in [\sigma\varepsilon, \mu_2\varepsilon]$ and, therefore, $\widetilde{X} < X^\star$. To summarize, when $w \in (\sigma\varepsilon, \mu_2\varepsilon)$:

- If $\xi \geq \tilde{\xi}$ we have $\widetilde{X} < \mathcal{X}$ when $w < w_a$ and $\widetilde{X} > \mathcal{X}$ when $w > w_a$.
- If $\xi < \tilde{\xi}$, we have $\widetilde{X} < \mathcal{X}$.

$(G.1)$

When $\mathcal{X} = \overline{X}$ ($w \geq \max\{\sigma\varepsilon, \mu_2\varepsilon\}$), we have: $\widetilde{X} - \overline{X} = [(\beta R e^\xi - 1)w - (e^\xi - 1)(R-1)\varepsilon]/[(1-\beta R)(e^\xi - 1)]$. Then, if $\xi < \hat{\xi} = \ln[1/(\beta R)]$ we have $\widetilde{X} < \overline{X}$. If $\xi > \hat{\xi}$, we have $\widetilde{X} < \overline{X}$ if and only if

$w < w_a = (e^{\xi} - 1)(R - 1)\varepsilon/[\beta R e^{\xi} - 1]$. Importantly, $w_a > \max\{\sigma\varepsilon, \mu_2\varepsilon\}$ if and only if $\hat{\xi} < \xi < \tilde{\xi}$. To summarize, when $w \geq \max\{\sigma\varepsilon, \mu_2\varepsilon\}$:

- If $\xi < \hat{\xi}$, we have $\widetilde{X} < \mathcal{X}$.
- If $\hat{\xi} < \xi < \tilde{\xi}$, we have $\widetilde{X} < \mathcal{X}$ when $w < w_a$ and $\widetilde{X} > \mathcal{X}$ when $w > w_a$. $\quad (G.2)$
- If $\xi > \tilde{\xi}$, we have $\mathcal{X} < \widetilde{X}$ for all w.

Then, according to $(G.1)$ and $(G.2)$, if the threshold $X^i = \mathcal{X}(w^{i-1})$ is reached (that is, if $\widetilde{X}(w^{i-1}) > \mathcal{X}(w^{i-1})$) then also the threshold X^{i+1} will be necessarily reached (i.e. $\widetilde{X}(w^i) > \mathcal{X}(w^i)$). By iterating the same argument, if there is a member of the dynasty whose wealth reaches the first threshold X^1 then the wealth accumulated by the following members will eventually also reach all the other thresholds. $\square$