Dynastic Accumulation of Wealth.

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Details of results of Appendix E.

To derive the comparative-static results of Appendix E we need to complete the characterization of the LRD, represented synthetically in Propositions 4 and 5, in term of the fundamental parameters (β, R, w, ξ) and initial wealth (X_0). To save space and many tedious calculations, instead of providing directly the whole characterization for each possible value of w we provide, separately for $\beta R > 1$ and $\beta R < 1$: (i) the type of LRD as a function of initial wealth X_0 and of the thresholds $\mathcal{X}$, $\widetilde{X}$ and $\widehat{X}$; and (ii) the conditions on the model parameters such that the relations found in point (i) are satisfied.

□ **Case where $\beta R > 1$:**

★ When $w < \eta\sigma\varepsilon$, the LRD is ZERO-WORK if $X_0 < \widehat{X}$ and INFI-RENT if $X_0 > \widehat{X}$.

★ When $\mu_1\varepsilon < w < \min\{\sigma\varepsilon, \mu_2\varepsilon\}$ and:

⇒ $X^\star < \widetilde{X}$, the LRD is ZERO-WORK if $X_0 < \widehat{X}$ and INFI-RENT if $X_0 > \widehat{X}$.

⇒ When $X^\star > \widetilde{X}$ and $\beta R[w + X^\star - \sigma\varepsilon] < \widehat{X}$, there exist some a_i, b_i, a'_i , b'_i, c_i, d_i, c'_i and d'_i with $a_i, b_i, a'_i, b'_i \in (\widetilde{X}, X^\star)$ and $c_i, d_i, c'_i, d'_i \in (\sigma\varepsilon, \widehat{X})$ such that the LRD is ZERO-WORK if $X_0 \in (0, \widetilde{X}) \cup \bigcup(a_i, b_i) \cup (X^\star, \sigma\varepsilon) \cup \bigcup(c_i, d_i)$, FINI-MIX if $X_0 \in \bigcup(a'_i, b'_i) \cup \bigcup(c'_i, d'_i)$ and INFI-RENT if $X_0 > \widehat{X}$.

⇒ When $X^\star > \widetilde{X}$ and $\beta R[w + X^\star - \sigma\varepsilon] > \widehat{X}$, there exist some a_i, b_i, a'_i , b'_i, a''_i, b''_i, c_i, d_i, c'_i, d'_i, c''_i and d''_i with $a_i, b_i, a'_i, b'_i, a''_i, b''_i \in (\widetilde{X}, X^\star)$ and $c_i, d_i, c'_i, d'_i, c''_i, d''_i \in (\sigma\varepsilon, \widehat{X})$ such that the LRD is ZERO-WORK if $X_0 \in (0, \widetilde{X}) \cup \bigcup(a_i, b_i) \cup (X^\star, \sigma\varepsilon) \cup \bigcup(c_i, d_i)$, FINI-MIX if $X_0 \in \bigcup(a'_i, b'_i) \cup \bigcup(c'_i, d'_i)$ and INFI-RENT if $X_0 \in \bigcup(a''_i, b''_i) \cup \bigcup(c''_i, d''_i) \cup (\widehat{X}, +\infty)$.

⊠ *Comparison between $X^\star$ and $\widetilde{X}$.*

⇒ When $R(e^\xi - 1) < \sigma$, we have $X^\star < \widetilde{X}$ for all $w \in (\mu_1\varepsilon, \min\{\sigma\varepsilon, \mu_2\varepsilon\})$.

⇒ When $R(e^\xi - 1) > \sigma$, there exists a $w_1 \in (\mu_1\varepsilon, \min\{\sigma\varepsilon, \mu_2\varepsilon\})$, where $X^\star(w_1) = \beta R(\sigma\varepsilon - w_1)/(\beta R - 1)$, such that, $X^\star < \widetilde{X}$ if $w \in (\mu_1\varepsilon, w_1)$ and $X^\star > \widetilde{X}$ if $w \in (w_1, \min\{\sigma\varepsilon, \mu_2\varepsilon\})$.

⊠ *Comparison between $\widehat{X}$ and $\beta R[w + X^\star - \sigma\varepsilon]$.*

Let $R_1 = (\sigma\varepsilon/X^\star(\sigma\varepsilon) + 1)/\beta$ if $e^\xi > 2 - \beta$ and $R_1 = 1/\beta + \sigma/(e^\xi - 1)$ if $e^\xi < 2 - \beta$:

$\Rightarrow$ When $R < R_1$, we have $\beta R[w + X^\star - \sigma\varepsilon] < \widehat{X}$ for all $w \in (\mu_1\varepsilon, \min\{\sigma\varepsilon, \mu_2\varepsilon\})$.

$\Rightarrow$ When $R > R_1$, there exists a unique $w_2 \in (\mu_1\varepsilon, \min\{\sigma\varepsilon, \mu_2\varepsilon\})$ satisfying $w_2 + X^\star(w_2) = \widehat{X}$ such that, $\beta R[w + X^\star - \sigma\varepsilon] < \widehat{X}$ if $w \in (\mu_1\varepsilon, w_2)$, and $\beta R[w + X^\star - \sigma\varepsilon] > \widehat{X}$ if $w \in (w_2, \min\{\sigma\varepsilon, \mu_2\varepsilon\})$.

★ When $\mu_2\varepsilon < w < \sigma\varepsilon$ and:

$\Rightarrow$ $\widetilde{X} < \widehat{X} < \overline{X}$, the LRD is ZERO-WORK if $X_0 < \widetilde{X}$ and INFI-RENT if $X_0 > \widetilde{X}$.

$\Rightarrow$ $\overline{X} < \widetilde{X} < \widehat{X}$, the LRD is ZERO-WORK if $X_0 < \widehat{X}$ and INFI-RENT if $X_0 > \widehat{X}$.

$\Rightarrow$ $\widetilde{X} < \overline{X} < \widehat{X}$, $\beta R[w + \overline{X} - \sigma\varepsilon] < \widehat{X}$ and $\beta R[\overline{X} - \sigma\varepsilon] < \widetilde{X}$, there exists some a_i, b_i, a_i' , b_i', c_i, d_i, c_i' and d_i' with $a_i, b_i, a_i', b_i' \in (\widetilde{X}, \overline{X})$ and $c_i, d_i, c_i', d_i' \in (\overline{X}, \widehat{X})$ such that the LRD is ZERO-WORK if $X_0 \in (0, \widetilde{X}) \cup \bigcup(a_i, b_i) \cup \bigcup(c_i, d_i)$, FINI-MIX if $X_0 \in \bigcup(a_i', b_i') \cup \bigcup(c_i', d_i')$ and INFI-RENT if $X_0 > \widehat{X}$.

$\Rightarrow$ $\widetilde{X} < \overline{X} < \widehat{X}$, $\beta R[w + \overline{X} - \sigma\varepsilon] < \widehat{X}$ and $\beta R[\overline{X} - \sigma\varepsilon] > \widetilde{X}$, the LRD is ZERO-WORK if $X_0 < \widetilde{X}$, FINI-MIX if $X_0 \in (\widetilde{X}, \widehat{X})$ and INFI-RENT if $X_0 > \widehat{X}$.

$\Rightarrow$ $\widetilde{X} < \overline{X} < \widehat{X}$, $\beta R[w + \overline{X} - \sigma\varepsilon] > \widehat{X}$ and $\beta R[\overline{X} - \sigma\varepsilon] < \widetilde{X}$, there exist some a_i, b_i, a_i' , b_i', a_i'', b_i'', c_i, d_i, c_i', d_i', c_i'' and d_i'' with $a_i, b_i, a_i', b_i', a_i'', b_i'' \in (\widetilde{X}, \overline{X})$ and $c_i, d_i, c_i', d_i', c_i'', d_i'' \in (\overline{X}, \widehat{X})$ such that the LRD is ZERO-WORK if $X_0 \in (0, \widetilde{X}) \cup \bigcup(a_i, b_i) \cup \bigcup(c_i, d_i)$, FINI-MIX if $X_0 \in \bigcup(a_i', b_i') \cup \bigcup(c_i', d_i')$ and INFI-RENT if $X_0 \in \bigcup(a_i'', b_i'') \cup \bigcup(c_i'', d_i'') \cup (\widehat{X}, +\infty)$.

$\Rightarrow$ $\widetilde{X} < \overline{X} < \widehat{X}$, $\beta R[w + \overline{X} - \sigma\varepsilon] > \widehat{X}$ and $\beta R[\overline{X} - \sigma\varepsilon] > \widetilde{X}$, there exists some a_i, b_i, a_i' , b_i', c_i, d_i, c_i' and d_i' with $a_i, b_i, a_i', b_i' \in (\widetilde{X}, \overline{X})$ and $c_i, d_i, c_i', d_i' \in (\overline{X}, \widehat{X})$ such that the LRD is ZERO-WORK if $X_0 \in (0, \widetilde{X})$, FINI-MIX if $X_0 \in \bigcup(a_i, b_i) \cup \bigcup(c_i, d_i)$ and INFI-RENT if $X_0 \in \bigcup(a_i', b_i') \cup \bigcup(c_i', d_i') \cup (\widehat{X}, +\infty)$.

⊠ *Comparison between* $\widetilde{X}$ *and* $\widehat{X}$.

$\Rightarrow$ We have $\widetilde{X} < \widehat{X}$ for all $w \in (\mu_2\varepsilon, \sigma\varepsilon)$

⊠ *Comparison between* $\widetilde{X}$ *and* $\overline{X}$.

$\Rightarrow$ When $R(e^\xi - 1) < \sigma$, there exists a $w_3 \in (\mu_2\varepsilon, \sigma\varepsilon)$, where $w_3 = \varepsilon(e^\xi - 1)(R - 1)/[\beta R e^\xi - 1]$, such that, $\overline{X} < \widetilde{X}$ if $w \in (\mu_2\varepsilon, w_3)$ and $\widetilde{X} < \overline{X}$ if $w \in (w_3, \sigma\varepsilon)$.

$\Rightarrow$ When $\sigma < R(e^\xi - 1) < R(1 - \beta)$, we have $\widetilde{X} < \overline{X}$ for all $w \in (\mu_2\varepsilon, \sigma\varepsilon)$.

⊠ *Comparison between* $\overline{X}$ *and* $\widehat{X}$.

$\Rightarrow$ When $e^\xi - 1 < \sigma(\beta R - 1)/(R - 1)$, there exists a $w_4 \in (\mu_2\varepsilon, \sigma\varepsilon)$, where $w_4 = (e^\xi - 1)(R - 1)\varepsilon/(\beta R - 1)$, such that, $\overline{X} < \widehat{X}$ if $w \in (\mu_2\varepsilon, w_4)$ and $\widehat{X} < \overline{X}$ if $w \in (w_4, \sigma\varepsilon)$.

$\Rightarrow$ When $\sigma(\beta R - 1)/(R - 1) < e^\xi - 1 < 1 - \beta$, we have $\overline{X} < \widehat{X}$ for all $w \in (\mu_2\varepsilon, \sigma\varepsilon)$.

⊠ *Comparison between* $\widehat{X}$ *and* $\beta R[w + \overline{X} - \sigma\varepsilon]$.

$\Rightarrow$ When $\big[1 < \beta R < 2$ and $e^\xi - 1 < \chi_1 = \sigma(\beta R - 1)/[1/\beta + \beta R - 2]\big]$or $\big[\beta R > 2$ and $e^\xi - 1 < \chi_2 =$

$(1-\beta)/(\beta R-1)]$, there exists a $w_5 \in (\mu_2\varepsilon, \sigma\varepsilon)$, where $w_5 = \varepsilon(e^\xi - 1)(R-1)/[e^\xi(\beta R - 1)]$, such that: $\beta R[w + \overline{X} - \sigma\varepsilon] < \widehat{X}$ if $w \in (\mu_2\varepsilon, w_5)$ and $\beta R[w + \overline{X} - \sigma\varepsilon] > \widehat{X}$ if $w \in (w_5, \sigma\varepsilon)$.

⇒ When $1 < \beta R < 2$ and $\chi_1 < e^\xi - 1 < 1 - \beta$, we have $\beta R[w + \overline{X} - \sigma\varepsilon] < \widehat{X}$ for all $w \in (\mu_2\varepsilon, \sigma\varepsilon)$.

⇒ When $\beta R > 2$ and $\chi_2 < e^\xi < 1 - \beta$, we have $\beta R[w + \overline{X} - \sigma\varepsilon] > \widehat{X}$ for all $w \in (\mu_2\varepsilon, \sigma\varepsilon)$.

⊠ *Comparison between $\widetilde{X}$ and $\mu R[\overline{X} - \sigma\varepsilon]$.*

⇒ There exists a $w_6 \in (\mu_2\varepsilon, \sigma\varepsilon)$, where $w_6 = \varepsilon(e^\xi - 1)(R-1)/(e^\xi + \beta R - 2)$, such that, $\beta R[\overline{X} - \sigma\varepsilon] < \widetilde{X}$ if $w \in (\mu_2\varepsilon, w_6)$ and $\beta R[\overline{X} - \sigma\varepsilon] > \widetilde{X}$ if $w \in (w_6, \sigma\varepsilon)$.

★ When $\sigma\varepsilon < w < \mu_2\varepsilon$ and:

⇒ $\beta R[w - \sigma\varepsilon] > \widehat{X}$, the LRD is INFI-RENT for all X_0.

⇒ $\beta R[w - \sigma\varepsilon] < \widehat{X}$ and $\beta R[w + X^\star - \sigma\varepsilon] > \widehat{X}$, there exist some a_i, b_i, a_i' , b_i', a_i'' and b_i'' with $a_i, b_i, a_i', b_i', a_i'', b_i'' \in (0, \widehat{X})$ such that the LRD is ZERO-MIX if $X_0 \in \bigcup(a_i, b_i)$, FINI-MIX if $X_0 \in \bigcup(a_i', b_i')$ and INFI-RENT if $X_0 \in \bigcup(a_i'', b_i'') \cup (\widehat{X}, +\infty)$.

⇒ $\sigma\varepsilon < \beta R[w + X^\star - \sigma\varepsilon] < \widehat{X}$, there exists some a_i, b_i, a_i' and b_i' with $a_i, b_i, a_i', b_i' \in (0, \widehat{X})$ such that the LRD is ZERO-MIX if $X_0 \in \bigcup(a_i, b_i)$, FINI-MIX if $X_0 \in \bigcup(a_i', b_i')$ and INFI-RENT if $X_0 > \widehat{X}$.

⇒ $\beta R[w + X^\star - \sigma\varepsilon] < \sigma\varepsilon$, the LRD is ZERO-MIX if $X_0 < \widehat{X}$ and INFI-RENT if $X_0 > \widehat{X}$.

⊠ *Comparison between $\widehat{X}$ and $\beta R[w - \sigma\varepsilon]$.*

⇒ When $2 - \beta < e^\xi < 1 + \beta R(1-\beta)/(\beta R - 1)$, we have $\beta R[w - \sigma\varepsilon] < \widehat{X}$ for all $w \in (\sigma\varepsilon, \mu_2\varepsilon)$.

⇒ When $e^\xi > 1 + \beta R(1-\beta)/(\beta R - 1)$, there exists a $w_7 \in (\sigma\varepsilon, \mu_2\varepsilon)$, where $w_7 = (1-\beta)R\varepsilon/(\beta R - 1)$, such that $\beta R[w - \sigma\varepsilon] < \widehat{X}$ if $w \in (\sigma\varepsilon, w_7)$ and $\beta R[w - \sigma\varepsilon] > \widehat{X}$ if $w \in (w_7, \mu_2\varepsilon)$.

⊠ *Comparison $\widehat{X}$ and $\beta R[w + X^\star - \sigma\varepsilon]$.*

⇒ When $1 < \beta R < 1 + \beta\sigma/(e^\xi - 1)$, we have $\beta R[w + X^\star - \sigma\varepsilon] < \widehat{X}$ for all $w \in (\sigma\varepsilon, \mu_2\varepsilon)$.

⇒ When $1 + \beta\sigma/(e^\xi - 1) < \beta R < 1 + \sigma\varepsilon/X^\star(\sigma\varepsilon)$, we have $w_2 \in (\sigma\varepsilon, \mu_2\varepsilon)$ and $\beta R[w + X^\star - \sigma\varepsilon] < \widehat{X}$ if $w \in (\sigma\varepsilon, w_2)$ and $\beta R[w + X^\star - \sigma\varepsilon] > \widehat{X}$ if $w \in (w_2, \mu_2\varepsilon)$.

⇒ When $\beta R > 1 + \sigma\varepsilon/X^\star(\sigma\varepsilon)$, we have $\beta R[w + X^\star - \sigma\varepsilon] > \widehat{X}$ for all $w \in (\sigma\varepsilon, \mu_2\varepsilon)$.

⊠ *Comparison $\sigma\varepsilon$ and $\beta R[w + X^\star - \sigma\varepsilon]$.*

⇒ When $1 < \beta R < \sigma\varepsilon/X^\star(\sigma\varepsilon)$, there exists a $w_8 \in (\sigma\varepsilon, \mu_2\varepsilon)$ satisfying $w_8 + X^\star(w_8) = (1 + \beta R)\sigma\varepsilon/(\beta R)$ such that $\beta R[w + X^\star - \sigma\varepsilon] < \sigma\varepsilon$ if $w \in (\sigma\varepsilon, w_8)$ and $\beta R[w + X^\star - \sigma\varepsilon] > \sigma\varepsilon$ if $w \in (w_8, \mu_2\varepsilon)$.

⇒ When $\beta R > \sigma\varepsilon/X^\star(\sigma\varepsilon)$, we have $\beta R[w + X^\star - \sigma\varepsilon] > \sigma\varepsilon$ for all $w \in (\sigma\varepsilon, \mu_2\varepsilon)$.

★ When $w > \max\{\sigma\varepsilon, \mu_2\varepsilon\}$ and:

⇒ $\widehat{X} < \overline{X}$ or when ($\overline{X} < \widehat{X}$ and $\beta R[w - \sigma\varepsilon] > \widehat{X}$), the LRD is INFI-RENT for all X_0.

⇒ $\overline{X} < \widehat{X}$, $\beta R[w - \sigma\varepsilon] < \widehat{X}$ and $\beta R[w + \overline{X} - \sigma\varepsilon] > \widehat{X}$, there exist some a_i, b_i, a'_i et b'_i avec $a_i, b_i, a'_i, b'_i \in (0, \widehat{X})$ such that the LRD is FINI-MIX if $X_0 \in \bigcup(a_i, b_i)$ and INFI-RENT if $X_0 \in \bigcup(a'_i, b'_i) \cup (\widehat{X}, +\infty)$.

⇒ $\overline{X} < \widehat{X}$ and $\beta R[w + \overline{X} - \sigma\varepsilon] < \widehat{X}$, the LRD is FINI-MIX if $X_0 < \widehat{X}$ and INFI-RENT if $X_0 > \widehat{X}$.

⊠ *Comparison between $\overline{X}$ and $\widehat{X}$.*

⇒ When $e^\xi < 1 + \sigma(\beta R - 1)/(R - 1)$, we have $\widehat{X} < \overline{X}$ for all $w \in (\max\{\sigma\varepsilon, \mu_2\varepsilon\}, +\infty)$.

⇒ When $\sigma(\beta R-1)/(R-1)+1 < e^\xi$, we have $w_4 > \max\{\sigma\varepsilon, \mu_2\varepsilon\}$ and $\overline{X} < \widehat{X}$ if $w \in (\max\{\sigma\varepsilon, \mu_2\varepsilon\}, w_4)$ and $\widehat{X} < \overline{X}$ if $w \in (w_4, +\infty)$.

⊠ *Comparison between $\widehat{X}$ and $\beta R[w + \overline{X} - \sigma\varepsilon]$.*

⇒ When $\left[1 < \beta R < 2 \text{ and } e^\xi - 1 \in (0, \chi_1) \cup (\chi_2, +\infty)\right]$ or $\left[\beta R > 2\right]$, we have $\beta R[w + \overline{X} - \sigma\varepsilon] > \widehat{X}$ for all $w \in (\max\{\sigma\varepsilon, \mu_2\}, +\infty)$.

⇒ When $1 < \beta R < 2$ and $\chi_1 < e^\xi - 1 < \chi_2$, we have $w_5 > \max\{\sigma\varepsilon, \mu_2\varepsilon\}$ and $\beta R[w + \overline{X} - \sigma\varepsilon] < \widehat{X}$ if $w \in (\max\{\sigma\varepsilon, \mu_2\varepsilon\}, w_5)$ and $\beta R[w + \overline{X} - \sigma\varepsilon] > \widehat{X}$ if $w \in (w_5, +\infty)$.

⊠ *Comparison between $\widehat{X}$ and $\beta R[w - \sigma\varepsilon]$.*

⇒ When $e^\xi - 1 < (1 - \beta)R/(\beta R - 1)$, we have $w_7 > \max\{\sigma\varepsilon, \mu_2\varepsilon\}$ and $\beta R[w - \sigma\varepsilon] < \widehat{X}$ if $w \in (\max\{\sigma\varepsilon, \mu_2\varepsilon\}, w_7)$ and $\beta R[w - \sigma\varepsilon] > \widehat{X}$ if $w \in (w_7, +\infty)$.

⇒ When $e^\xi - 1 > (1 - \beta)R/(\beta R - 1)$, we have $\beta R[w - \sigma\varepsilon] > \widehat{X}$ for all $w \in (\max\{\sigma\varepsilon, \mu_2\varepsilon\}, +\infty)$.

□ **Case where $\beta R < 1$:**

★ When $w < \sigma\varepsilon$, the LRD is ZERO-WORK for all X_0.

★ When $\sigma\varepsilon < w < \mu_2\varepsilon$ and:

⇒ $\widetilde{X} < X^\star$, the LRD is FINI-WORK for all X_0.

⇒ $X^\star < \widetilde{X} < \sigma\varepsilon$, the LRD is ZERO-MIX for all X_0.

⇒ $\beta R[w + X^\star - \sigma\varepsilon] < \sigma\varepsilon < \widetilde{X}$, the LRD is ZERO-MIX for all X_0.

⇒ $\sigma\varepsilon < \widetilde{X}$ and $\beta R[w + X^\star - \sigma\varepsilon] > \sigma\varepsilon$, there exist some a_i, b_i, a'_i and $b'i$ such that ZERO-MIX if $X_0 \in \bigcup(a_i, b_i)$ and FINI-MIX if $X_0 \in \bigcup(a'_i, b'_i)$.

⊠ *Comparison between $X^\star$ and $\sigma\varepsilon$.*

⇒ We have $X^\star < \sigma\varepsilon$ for all $w \in (\sigma\varepsilon, \mu_2\varepsilon)$.

⊠ *Comparison between $\widetilde{X}$ and $\sigma\varepsilon$.*

⇒ When $e^\xi < 1 + \sigma/R$, we have $\widetilde{X} < \sigma\varepsilon$ for all $w \in (\sigma\varepsilon, \mu_2\varepsilon)$.

⇒ When $e^\xi > 1 + \sigma/R$, we have $\widetilde{X} < \sigma\varepsilon$ if $w \in (\sigma\varepsilon, \sigma\varepsilon/(\beta R))$ and $\widetilde{X} > \sigma\varepsilon$ if $w \in (\sigma\varepsilon/(\beta R), \mu_2\varepsilon)$.

⊠ *Comparison between $X^\star$ and $\widetilde{X}$.*

⇒ When $e^\xi < 1 + \sigma/R$, we have $\widetilde{X} < X^\star$ for all $w \in (\sigma\varepsilon, \mu_2\varepsilon)$.

⇒ When $e^\xi > 1 + \sigma/R$, we have $w_1 \in (\sigma\varepsilon, \mu_2\varepsilon)$ and $\widetilde{X} < X^\star$ if $w \in (\sigma\varepsilon, w_x)$ and $\widetilde{X} > X^\star$ if $w \in (w_{xx}, \mu_2\varepsilon)$.

⊠ *Comparison between $\sigma\varepsilon$ and $\beta R[w + X^\star - \sigma\varepsilon]$.*

⇒ When $\beta R < \sigma/\mu_2$, we have $\beta R[w + X^\star - \sigma\varepsilon] < \sigma\varepsilon$ for all $w \in (\sigma\varepsilon, \mu_2\varepsilon)$.

⇒ When $\sigma/\mu_2 < \beta R < \sigma\varepsilon/X^\star(\sigma\varepsilon)$, we have $w_8 \in (\sigma\varepsilon, \mu_2\varepsilon)$ and $\beta R[w+X^\star-\sigma\varepsilon] < \sigma\varepsilon$ if $w \in (\sigma\varepsilon, w_8)$ and $\beta R[w + X^\star - \sigma\varepsilon] > \sigma\varepsilon$ if $w \in (w_8, \mu_2\varepsilon)$.

⇒ When $\beta R > \sigma\varepsilon/X^\star(\sigma\varepsilon)$, we have $\sigma\varepsilon < \beta R[w + X^\star - \sigma\varepsilon]$ for all $w \in (\sigma\varepsilon, \mu_2\varepsilon)$.

★ When $w > \max\{\sigma\varepsilon, \mu_2\varepsilon\}$ and:

⇒ $\overline{X} < \widetilde{X}$, the LRD is FINI-MIX for all X_0.

⇒ $\widetilde{X} < \overline{X}$, the LRD is FINI-WORK for all X_0.

⊠ *Comparison $\overline{X}$ and $\widetilde{X}$.*

⇒ When $e^\xi < 1/(\beta R)$, we have $\widetilde{X} < \overline{X}$ for all $w \in (\max\{\sigma\varepsilon, \mu_2\varepsilon\}, +\infty)$.

⇒ When $1/(\beta R) < e^\xi < 1+\sigma/R$, we have $w_3 > \max\{\sigma\varepsilon, \mu_2\varepsilon\}$ and $\widetilde{X} < \overline{X}$ if $w \in (\max\{\sigma\varepsilon, \mu_2\varepsilon\}, w_3)$ and $\widetilde{X} > \overline{X}$ if $w \in (w_3, +\infty)$.

⇒ When $e^\xi > 1 + \sigma/R$, we have $\widetilde{X} > \overline{X}$ for all $w \in \max\{\sigma\varepsilon, \mu_2\varepsilon\}, +\infty)$.

Figures 10 to 12 restrict the analysis to $w > \sigma\varepsilon$ (i.e. to the two cases, $\sigma\varepsilon < w < \mu_2\varepsilon$ and $w > \max\{\sigma\varepsilon, \mu_2\varepsilon\}$). Figures 13 to 15 focus on cases where $X_0 = 0$ while leaving w unrestricted.