Family Size, Investment in Daughters, and the Politics of Gender Equality

Preliminary and incomplete draft of work in progress

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It is widely known that there is a negative correlation between fertility and gender equality. Mechanisms by which gender equality can lead to reduced fertility are numerous. For example, women who have opportunities for education and employment gain greater control over their fertility, either due to more equal power relationships within marriages or because of increased knowledge of birth control options. Thus, it is commonly argued that gender equality is a major cause of reductions in fertility (see, e.g., Sen 1999).

This paper examines the possibility that causation may also run in the other direction, i.e., that reduced fertility may lead to increased equality. The intuition of the argument is as follows. Consider a relatively-poor society, in which parents allocate resources unequally among children and the level of economic opportunity for women is set via a political process controlled by men. An exogenous decrease in family size could lead to increased equality, via two mutually-reinforcing mechanisms. The first is an investment effect: as family size decreases, parents increase the amount of resources allocated to daughters, particularly in families with few sons. The second is a political effect: parents who have few sons and who invest in daughters will support increased equality of economic opportunity. Allendorf (2012), as part of an ethnographic analysis of gender in South Asian villages, develops a verbal theory in which smaller families, especially those that only have daughters, invest more in girls and change their expectations about the appropriate gender roles in society. But to the best of my knowledge this argument has not been formalized.

There are, of course, other possible explanations for how decreased fertility could lead to increased gender equality. The most direct explanation is that if women have a large number of children and few resources (government or private) for childcare, they will find it difficult to participate in the formal economy. If parents anticipate that their daughters will have careers instead of, or in addition to, bearing and raising children, they will invest more in their daughters. Note that this explanation focuses on how intra-household constraints on time allocation affect women's ability to take advantage of opportunities in the labor market. It does not, however, explain how changes in family size affect the political choice of government policies that determine equality of economic opportunity for women. Huber (1976) takes a step in that direction by developing a theoretical argument that decreased fertility leads to the formation of social movements for change by women who object to unequal treatment.[1]

The most similar papers to mine are ones that analyze women's property rights within marriage (Doepke and Tertilt 2009, and Fernandez 2014). There are several important ways that those papers differ from my model. First, they focus on property rights, whereas I focus on equality of opportunity in the economy and assume that women already have secure property rights (both in the sense that they can own property and in the sense that if married they are not property of their husbands). Second, in their models, property rights are discrete–i.e., women have either no rights or the same rights as men–whereas I allow for a continuum of levels of equality. Third, the driving force for why men may support equality differs. In Doepke and Tertilt, a man sees laws giving women increased rights within families as a means to induce parents of his children's future spouses to invest more in their human capital. In Fernandez, a man wants to avoid an implicit tax imposed by future husbands of his daughters. In my model, a father who has few sons cares more about the success of his daughters and thus becomes more supportive of equal opportunity.

In this paper, I first analyze political choice of equality of opportunity, and its links to family size, in a simple baseline model in which fertility and investment in children are exogenous. I then extend the model in several directions: by allowing for the possibility that men (and their sons) dislike economic competition from women who are granted employment rights, by endogenizing fertility as

[1]Other analyses of causes of gender inequality include Ross (2008), who argues that in Middle Eastern countries, oil production has kept women out of the workforce, leading to larger family sizes, patriarchal cultures, and lower political influence, as well as Alesina, Giuliano, and Nunn (2013), who show that countries that had ancient plough-based agriculture have lower levels of equality within households and in society.

a stopping problem, and by analyzing what happens if patterns of investment in children change as fertility declines.

Baseline Model

The key variable of interest in the model is $e \in [0, 1]$, which represents government policies that affect women's economic opportunities. At one extreme, $e \approx 0$ means that women are barred from participation in the formal economy outside the household. Increases in e would represent (in order) the following types of policies: (1) Allowing women to participate in the economy but prohibiting them from holding high-pay, high-status jobs, (2) Formal legal equality of opportunity, (3) Effective enforcement of equality of opportunity, and (4) Policies that decouple the bearing and rearing of children and thereby level the playing field for men and women to participate in the labor market, e.g., paid leave for all parents or the right to return to a job track after childbearing. My model is not intended to apply to societies at the highest levels of e. Rather, the empirical domain is societies that are patriarchal, and in which the key issue at stake is whether women are allowed to hold jobs and whether they have some form of legal equality of opportunity.

In the model, political power is held by the men, or a subset of the men that I refer to as patriarchs. Within the set of patriarchs, e is chosen via majority rule (though note that this could be majority rule within a powerful elite rather than all men in society).

I begin with a simple baseline model, in which investment in children is exogenous and the number of children in each household is exogenously set at an even number $2K$. For family i, order the household's children as $1, ..., 2K$, starting with the boys then girls. Note that this is not birth order, but rather a ranking of their importance within the household. Given his household composition, patriarch i's utility given societal equality of opportunity e depends on his direct disutility from e as well as the economic success of his children:

$$U_i(e) = D(e) + \sum_{k=1}^{2K} \delta^{k-1} y_{i_k} \text{ where } y_{i_k} = \begin{cases} 1 \text{ if boy} \\ \gamma e \text{ if girl} \end{cases}. \quad (1)$$

In Equation 1, $\delta^{k-1} y_{i_k}$ is the utility that patriarch i gets from the economic output of his k'th ranked child, where, $\delta \in [0,1]$. The parameter $\gamma \in [0,1]$ specifies how much a patriarch values success of a k'th ranked child who is a daughter relative to a k'th ranked child who is a son. The patriarch also gets direct disutility $D(e)$ from equality of opportunity. I now discuss several aspects of this utility function in more detail.

Discounting of lower-ranked children I assume that patriarchs place disproportionate weight on their most successful children, as parametrized by $\delta \in [0,1]$. A society with high δ is approximately egalitarian within families, in the sense that a patriarch equally values the success of all of his children. In contrast, a society with δ close to zero is one in which the patriarch cares almost exclusively about the success of the highest-ranked children. There are several reasons that a patriarch may overweight the success of one child or a small number of his children. He might care only about the success of his most-successful child, who will inherit the family's wealth, carry on the family name, or care for him in his old age. However, even a patriarch who only cares about his most successful child knows that each child has some probability of dying, proving to be incompetent, or renouncing his or her affiliation with the family, so the patriarch would put some weight (represented by δ) on the success of lower-ranked children, who might need to step into that role. A more prosaic explanation for why patriarchs discount lower-ranked children is that parents like to brag about their children's greatest successes.[2]

[2] Note that the model, which is not intended to apply to wealthy modern countries, does not allow for behavior that is intra-family Rawlsian, in the sense of devoting extra resources to the least-successful children.

Devaluing the economic success of daughters In addition to placing higher emphasis on his most successful children, a patriarch may also get less utility from the success of a k'th ranked child who is a girl than from a k'th ranked child who is a boy. There are many possible reasons for this. One is simply a matter of taste, in societies where success of sons is valued more than success of daughters. Another reason is discrimination against women in the private job market, which makes it harder for them to attain economic success. More subtly, in some cultures, a daughter's "success" in a society may depend primarily on the family that she marries into, rather than her own economic output.

No explicit marriage or coupling In the model, a patriarch gets utility directly from his children's economic success, and I don't model marriage. By making this assumption, I do not mean to assume away the possibility of marriage. Rather, I follow Iversen and Rosenbluth (2011), who analyze intra-household bargaining and argue that outcomes for women who marry depend on their outside options, which in turn depend on their economic opportunities. I also assume that women have property rights within marriage, and do not become property of their husbands. For an analysis of property rights and legal rights within marriage, see Doepke and Tertilt (2009) and Fernandez (2014).

Patriarchs' disutility from equality of opportunity In the model, a patriarch not only cares about outcomes for his own children, but also cares directly about public policy, getting disutility $D(e)$ based on the societal equality of opportunity. This disutility could come from his own decreased opportunities in a labor market when he has to compete with women, or it could simply be a matter of bias and taste. To ensure interior solutions for e, I assume that $D(0) = D'(0) = 0, D'' < 0$, and $D'(1) = -\infty$.

I now analyze the choice of e in the baseline model.

Choice of e

Suppose that patriarch i's $2K$ children are g_i girls and $2K - g_i$ boys. Patriarch i's utility in Equation 1 is a strictly concave function of e, maximized at his induced ideal point, which is e_i such that

$$D'(e_i) = -\sum_{k=2K-g_i+1}^{2K} \delta^{k-1}\gamma.$$

Thus, in the model, having daughters–specifically a higher g_i holding family size $2K$ fixed–increases fathers' support for equality of opportunity. This is broadly consistent with evidence on the behavior of political elites, at least in the United States (Washington 2008, Glynn and Sen 2015).[3]

Under majority rule voting, the policy outcome e is the median of the patriarchs' ideal levels e_i. I assume that the number of patriarchs is sufficiently large, so that with an even family size $2K$ the median patriarch is always one who has K boys and K girls. Thus the level of equality of opportunity collectively chosen by the patriarchs, $e(K)$, solves

$$D'(e) = -\sum_{k=K+1}^{2K} \delta^{k-1}\gamma. \tag{2}$$

The comparative statics for $e(K)$ are intuitive. It is strictly increasing in γ, which measures the degree to which (holding a child's rank constant) a patriarch values the success of a daughter as much as the success as of a son. It is also strictly increasing in intra-household egalitarianism δ, because the lower-ranked children in a family are daughters.

To determine how an exogenous decrease in family size affects equality of opportunity, I compare

[3]For ordinary citizens the evidence is mixed. Warner and Steel (1999) find that fathers of daughters are more supportive of public policies that improve gender equity. Oswald and Powdthavee (2010) find that having daughters leads to a general liberal political orientation whereas Lee and Conley (2015) find null results. Shafer and Malhotra (2011) find that having daughters reduces men's support for traditional gender roles.

$e(K)$ and $e(K-1)$. From Equation 2, $e(K-1) > e(K)$ iff

$$\begin{aligned} \sum_{k=K}^{2K-2} \delta^{k-1}\gamma &> \sum_{k=K+1}^{2K} \delta^{k-1}\gamma \\ \delta^{K-1} &> \delta^{2K-2} + \delta^{2K-1} \\ 1 &> \delta^{K-1}(1+\delta). \end{aligned} \tag{3}$$

Thus, decreasing family size results in an increase in e iff δ is less than the unique $\delta(K)$ that solves Equation 3 with equality. We can also fix δ and allow K to vary. Note that the right hand side of Equation 3 is strictly decreasing in K, so there are two possibilities. If the society is sufficiently intra-family egalitarian (δ greater than $\delta(2) \approx 0.62$), then if K is sufficiently small, a decrease in family size causes e to decrease, but for larger values of K a decrease in family size causes e to increase. On the other hand, if the society is sufficiently intra-family inegalitarian (δ less than $\delta(2)$) then a decrease in family size increases e regardless of the value of K.

The following proposition summarizes the result for the baseline model.

Proposition 1 ***(Baseline)*** *A decrease in family size leads to increased equality of economic opportunity iff the society is sufficiently intra-family inegalitarian, i.e.,* $1 > \delta^{K-1}(1+\delta)$.

Extensions

Before analyzing endogenous fertility, I consider simple extensions and interpretations of the model.

Labor market and status competition Historically, opposition to equality has often come from men who dislike labor market and status competition from women. The baseline model can incorporate one aspect of such competition, because $D(e)$ can include effects on a patriarch's wages. However, the specification in the baseline cannot include effects on the wages of his sons. For this, I adopt a reduced form, in which sons' wages are deflated by $B(e)$, which is continuously differentiable, decreasing, strictly concave, and has $B(0) = 1$, $B'(0) = 0$, and $B'(1) > -1$. This last assumption means that even at full equality of opportunity, the marginal gains for women exceed the losses for men. More broadly, $B(e)$ could represent status competition, i.e., a patriarch's concern that, as women gain higher levels of equality of opportunity, his own sons (who he cares about more than his daughters) will not be as high-status within society.

Taking into account effects on his sons, the patriarch's utility is

$$U_i(e) = D(e) + \sum_{k=1}^{2K} \delta^{k-1} y_{i_k} \text{ where } y_{i_k} = \begin{cases} B(e) & \text{if boy} \\ \gamma e & \text{if girl} \end{cases}.$$

Incorporating effects on sons reinforces the effect that smaller families lead to higher levels of e, in the sense that this result holds for a wider range of δ than in the baseline model. Specifically, instead of Equation 2, the first order condition for a family with K boys and K girls is

$$D'(e) = -B'(e) \sum_{k=1}^{K} \delta^{k-1} - \sum_{k=K+1}^{2K} \delta^{k-1} \gamma. \tag{4}$$

Unsurprisingly, the level of equality is strictly less than it was in Equation 2, which didn't take into account patriarchs' concern about effects on their sons' wages and status.

For my purposes, what matters is how labor market competition influences the effect of family size on equality of opportunity. From Equation 4 we see that $e(K-1) > e(K)$ iff

$$\begin{aligned} B'(e) \sum_{k=1}^{K-1} \delta^{k-1} + \sum_{k=K}^{2K-2} \delta^{k-1} \gamma \quad &> \quad B'(e) \sum_{k=1}^{K} \delta^{k-1} + \sum_{k=K+1}^{2K} \delta^{k-1} \gamma \\ \delta^{K-1} \quad &> \quad \delta^{2K-2} + \delta^{2K-1} + \frac{B'(e)\,\delta^{K-1}}{\gamma} \end{aligned} \tag{5}$$

Because $B'(e) < 0$, the last term of Equation 5 is strictly negative, so comparing it with the expression in the penultimate line of Equation 3 we see that reductions in family size lead to increased equality of opportunity for a wider range of K and δ. The intuition is straightforward: with a smaller family, there are fewer sons who are harmed by a higher e, and whose interests are weighed against the interests of daughters who benefit from a higher e.

Proposition 2 *If patriarchs are concerned about labor market and status competition for their sons, then reductions in fertility lead to increased economic opportunity for women for a wider range of parameters than in the baseline model, i.e., the set* $\{\delta | e(K-1) > e(K)\}$ *is a strict superset of the values of* δ *for which equality increases as fertility decreases in the baseline model.*

Elites with large (or small) families In the model, the patriarchs are the politically powerful men in a society, so it is important to note that in societies with concentrated political power the level of equality of opportunity predicted by the model is not the level preferred by a typical male whose children are evenly divided between sons and daughters, but rather the level preferred by an elite male whose children are evenly divided between sons and daughters.

The relationship between wealth, power, and family size is quite complicated and has changed over time as societies have developed. Before the demographic transition, wealth and family size were positively correlated in most parts of the world (Vogl 2016). And anecdotal evidence suggests that in many countries with concentrated power, the families of political elites are substantially larger than the families of their subjects.

For some such societies (specifically, those that are intra-family inegalitarian), the model thus predicts that government policy on equality of opportunity lags behind the preferences of typical men in society. If, however, the demographic transition begins with the wealthy and then moves down to the masses, the elite-chosen level of equality of opportunity could be higher than what is preferred by most men.

Endogenous Fertility

I now extend the model to endogenize family size. The most straightforward way to do this would be to have households choose $2K$, and then have nature determine the number of boys and girls. However, it is more realistic to model family size as a stopping problem in which a family pays a cost c for each additional child. For now, I assume that the gender of the child cannot be controlled: nature determines whether it is a boy or a girl, each with probability 1/2. I also assume that all births are singletons.

The parameter c is the *net* cost of a child, which may depend a wide range of factors, including availability of contraception, government provision of education, and the possibility of informal child labor. The key factor that is not included in c is the benefit that parents get from their children's ultimate participation in the formal economy.

Given equality of opportunity e, Equation 1 implies that a household with b boys and g girls gets the following expected utility from an additional child

$$\frac{1}{2}\delta^{b+g}\gamma e + \frac{1}{2}\left(\delta^{b}\left(1-\gamma e\right) + \delta^{b+g}\gamma e\right) - c. \tag{6}$$

The first term is the utility from having an additional girl, who can be thought of as being the $g+b+1$ ranked child, and who is discounted by δ^{b+g} due to rank and by γe due to gender. The second term is the utility from having an additional boy, which can be thought of as changing the $b+1$ ranked child from a girl to a boy and adding an additional $g+b+1$ ranked child who is a girl.

Thus in the stopping problem the set of stable family sizes given e is

$$\hat{S}\left(e;c\right) \equiv \left\{\left(b,g\right) \text{ such that } \frac{1}{2}\delta^{b+g}\gamma e + \frac{1}{2}\left(\delta^{b}\left(1-\gamma e\right) + \delta^{b+g}\gamma e\right) > c\right\}.$$

Family size (b, g) can only be reached if a family with one less girl or a family with one less boy doesn't stop. I denote the set of family sizes that are stable and are reached by some families as

$$S\left(e;c\right) \equiv \left\{\left(b,g\right) \mid \left(b,g\right) \in \hat{S}\left(e;c\right) \text{ and } \left(b-1,g\right) \notin \hat{S}\left(e;c\right) \text{ or } \left(b,g-1\right) \notin \hat{S}\left(e;c\right)\right\}.$$

With this setup, the model makes predictions about family size and composition that are consistent with the theoretical and empirical demography literature (Yamaguchi 1989, Basu and de Jong 2010). Equation 6 implies that, holding fixed the total number of children $b+g$, the incentive to have another child is decreasing in the number of boys. Thus, girls are more likely than boys to come from large families. Also, compared to boys, girls typically have more younger siblings and fewer older siblings.

For my purposes, the key question is what level of equality of opportunity e will be chosen given families' stopping choices. Let $e(b,g)$ denote the level of equality when the median patriarch has b boys and g girls. Similarly to Equation 2 from the baseline model the first order condition for e is

$$D'(e) = -\sum_{k=b+1}^{b+g} \delta^{k-1}\gamma \tag{7}$$

In the baseline model, I assumed that all families have the same (even) number of children, which simplified the analysis of how family size affects e, because the median patriarch was always well-defined, with an equal number of boys and girls. With endogenous fertility, this is not guaranteed to be the case. Most obviously, it is possible that half of the families have strictly more boys than girls and half have strictly more girls than boys. For example, this would be the case if each family has exactly three children and then stops regardless of their genders.

More subtly, even if a family with equal size is stable, i.e., $b = g$ and $(b,g) \in S(e;c)$, this does not guarantee that the median patriarch is uniquely defined, and has an equal number of boys and girls. The reason for this is that it is possible that only families with g girls and $b-1$ boys choose to have another child, whereas families with $g-1$ girls and b boys stop. For example, a family with one child who is a boy may stop while a family with one child who is a girl may decide to have another child and then stop, regardless of the second child's gender.

Thus the model presents two technical difficulties. The first is non-uniqueness of the median e_i. This is not a major concern, because the natural sex ratio at birth is a bit greater than 1 boy to 1 girl. For example, if with a 1:1 sex ratio exactly half of the patriarchs have at least 2 boys and at most 1 girl and the other half of the patriarchs have at least 2 girls and at most 1 boy, then patriarchs with $b = 2$ and $g = 1$ would be the unique median if the sex ratio is slightly above 1:1.

The second difficulty is an integer problem, namely that changes in c may determine whether the equilibrium is one in which the median e_i is for a family with equal numbers of boys and girls or one in which the pivotal patriarch has one more boy than girl. This can create local nonmonotonicities. For example, suppose c increases from a level where a patriarch with $b = g = 3$ is the median to a level where a patriarch with $b = 3$ and $g = 2$ is pivotal. From Equation 7, we see that this would cause a decrease in e, i.e., $e\left(3,2\right) < e\left(3,3\right)$, because the median patriarch now has one less girl. Next, suppose c increases further to a level where a patriarch with $b = g = 2$ is the median. This would cause an increase in e, i.e., $e\left(2,2\right) > e\left(3,2\right)$

To deal with this second problem, I focus on changes *within* the set of parameters such that the median patriarch has $b = g$ and *within* the set of parameters such that the median patriarch has $b = g + 1$. This is a somewhat-awkward approach, but it makes it possible to analyze the overall effects of major changes in family size rather than focusing on local nonmonotonicities.

I first make an observation about the relationship between costs and pivotal family sizes: holding e fixed, for any cost c there is a unique pivotal family size with either $b = g$ or $b = g + 1$, and higher c's have smaller pivotal family sizes. Most details of the proof are in the appendix.

Observation 1 *Given e, there exist cutpoints $c_{0,0}\left(e\right), c_{1,0}\left(e\right), c_{1,1}\left(e\right), c_{2,1}\left(e\right)....$ such that $c_{n,n} > c_{n+1,n} > c_{n+1,n+1} > 0$ and if $c \in \left(c_{n+1,n}\left(e\right), c_{n,n}\left(e\right)\right)$ then the pivotal family size is $b = n + 1$ and $g = n$, whereas if $c \in \left(c_{n,n}\left(e\right), c_{n-1,n}\left(e\right)\right)$ then the pivotal family size is $b = g = n$.*

I define an *equilibrium* for the model as satisfying the following conditions. First, in the stopping problem, given equality of opportunity $e\left(b,g\right)$ each household has children until it reaches a stable family configuration, $\left(b,g\right) \in S\left(e\left(b,g\right);c\right)$. Second, equality is chosen according to the preferences of the median patriarch, i.e., given that the median has family structure $\left(b,g\right)$ the level of equality chosen is $e\left(b,g\right)$ from the first order condition in Equation 7, $D'\left(e\right) = -\sum_{k=b+1}^{b+g} \delta^{k-1}\gamma$.

Observation 1 implies that a particular $\left(b,g\right)$ with $b = g$ or $b = g+1$ is pivotal for a limited range of values of c, using the cutpoints $c_{b,g}\left(e\left(b,g\right)\right)$ generated by the level of equality $e\left(b,g\right)$ preferred by a patriarch with family structure (b, g). If c is above this range, this cannot be an equilibrium, because given societal equality of opportunity $e(b, g)$ families would stop having children before reaching (b, g).

I have not yet been able to fully characterize conditions for equilibrium existence. And (frustratingly), I have not been able to show that increasing c always leads to smaller family sizes. To show this is non-trivial, because $e(b, g)$ depends on the family size and the cutpoints $c_{b,g}(e)$ in Observation 1 depend on e. I haven't been able to rule out the seemingly-strange possibility that increasing c could eliminate an equilibrium with pivotal family size (b, g) and replace it with an equilibrium with a larger pivotal family size, e.g., $(b+1, g+1)$ with $e(b+1, g+1)$ that is substantially different from $e(b, g)$ and such that the new higher $c \in (c_{b+1,g}(e(b+1, g+1)), c_{b,g}(e(b+1, g+1)))$. For now, what I can show is the following simpler result.

Proposition 3 ***(Endogenous Family Size)*** *If an increase in c leads to a reduction in the equilibrium pivotal family size, from (b, g) to $(b-1, g-1)$, then economic opportunity increases iff the society is sufficiently intra-family inegalitarian, i.e., $1 > \delta^{b-1}(1+\delta)$.*

Note that this result holds regardless of whether $b = g$ or $b = g + 1$, so the logic of Proposition 1 continues to hold even if family sizes are endogenously-determined and also if the pivotal family size does not have the same number of boys and girls.

Gender selection It is also possible that people may choose not only the number of children, but also their gender. Historically, this required infanticide, but the modern method is an ultrasound followed by gender-selective abortion. Given that in some regions the sex ratio at birth is substantially higher than 1:1, this is clearly a large-scale phenomenon. However, gender selection typically is not practiced for first births, but rather by families that already have one or more daughters and wish to have a son.

Gender selection could be incorporated in my model of endogenous fertility, e.g., by allowing a family to pay additional cost c_s to select the gender of a child, in addition to the cost c of having a child. I conjecture that for reasonable parameter values, this would not affect the equilibrium level of equality, because gender selection would only occur for families that satisfy two conditions: being close to a stopping point for the number of children to have, and already having more girls than boys. Even if such a family chooses to have a boy, the patriarch will have an optimal e_i that is higher than the median preference (which is for a patriarch with $b = g$ or $b = g + 1$). The only way gender selection can affect the median, and hence the equilibrium level of equality of opportunity, is if c_s is so low that families in which the number of boys is already greater than or equal to the number of girls engage in gender selection to have yet another son. The most plausible situation where this could occur is if c is so high that a family will only have one child; in that circumstance, paying c_s to ensure birth of a son would be particularly appealing.

Variation in Investment

In this section, I return to the baseline model, with exogenous fertility and family size $2K$, but allow families to vary their investments across children. Specifically, family i invests x_{i_k} in its k'th ranked child, and patriarch i's utility is

$$U_i\left(e; x_{i_1}, ..., x_{i_{2K}}\right) = D\left(e\right) + \sum_{k=1}^{2K} \delta^{k-1} y_{i_k} - I\left(x_{i_1}, ..., x_{i_{2K}}\right) \text{ where } y_{i_k} = \begin{cases} x_{i_k} & \text{if boy} \\ \gamma e x_{i_k} & \text{if girl} \end{cases}.$$

The investment x_{i_k} can represent a wide range of scarce resources, such as money, time, food, schooling, or medical care. I don't assume a specific functional form for $I\left(x_{i_1}, ..., x_{i_{2K}}\right)$, which specifies the costs of investment and parents' tradeoff between their own consumption and investment. The total investment, as well as the allocation across children within a family could obviously vary as a function of both the number and gender of the children. I do, however assume that a family will invest more in the children who have the best economic prospects, as in Becker and Tomes (1976).[4] This means that within a family, investment in boys will be higher than investment in girls, both due to discounting as parametrized by δ and due to the fact that $\gamma \leq 1$ and $e < 1$. Note that lower investment in girls can arise either from parents placing a lower weight on daughters than sons $(\gamma < 1)$ or from the lack of equal opportunities for women in the economy $(e < 1)$. The fact that $\delta < 1$ also implies unequal investment within gender, so that $x_{i_k} > x_{i_{k+1}}, \forall k$.

At this point, I still have to complete several steps in the argument. First, I need to prove that the pivotal family continues has an equal number of boys and girls, which should be straightforward. I also need to take into account the fact that investments affect the choice of e, which in turn affects investments by determining economic opportunities for girls and women. It is not obvious that the equilibrium is unique, because high investment in girls makes a high e more appealing and a high e makes investment in girls more appealing. In this draft, I set these issues aside, and focus on how patterns of investment affect the relationship between family size and the politically chosen level of equality.

Suppose that in a family with K boys and K girls, investments are $(x_{m_1}, ..., x_{m_{2K}})$. Then, by the same reasoning as in Equation 2 the first order condition for $e(K)$ is

$$D'(e) = -\sum_{k=K+1}^{2K} \delta^{k-1} \gamma x_{m_k}.$$

Likewise, suppose that in a family with $K-1$ boys and $K-1$ girls, investments are $\left(\hat{x}_{m_1}, ..., \hat{x}_{m_{2K-2}}\right)$. The first order condition for $e(K-1)$ is

$$D'(e) = -\sum_{k=K}^{2K-2} \delta^{k-1} \gamma \hat{x}_{m_k}.$$

[4] In their model, variation is due to children's innate ability not societal opportunities.

We then see that $e(K-1) > e(K)$ iff

$$\begin{aligned}\sum_{k=K}^{2K-2} \delta^{k-1}\gamma\hat{x}_{m_k} &> \sum_{k=K+1}^{2K} \delta^{k-1}\gamma x_{m_k} \\ \delta^{K-1}\hat{x}_{m_K} + \sum_{k=K+1}^{2K-2} \delta^{k-1}\hat{x}_{m_k} &> \sum_{k=K+1}^{2K-2} \delta^{k-1}x_{m_k} + \delta^{2K-2}x_{m_{2K-1}} + \delta^{2K-1}x_{m_{2K}}. \qquad (8)\end{aligned}$$

This is similar to Equation 3 from the baseline model, in which a reduction in family size leads to increased equality iff $\delta^{K-1} > \delta^{2K-2} + \delta^{2K-1}$. However there are a few differences. The terms $\sum_{k=K+1}^{2K-2} \delta^{k-1}\hat{x}_{m_k}$ and $\sum_{k=K+1}^{2K-2} \delta^{k-1}x_{m_k}$ were not present in the baseline model, in which investment was implicitly held constant for all children. The other difference is that whereas the baseline model's condition was $\delta^{K-1} > \delta^{2K-2} + \delta^{2K-1}$, these terms are now multiplied by investments, $\hat{x}_{m_K}$, $x_{m_{2K-1}}$, and $x_{m_{2K}}$.

Intuitively, we should expect that larger families will invest less in each child, i.e., $\hat{x}_{m_k} > x_{m_k}$. This effect is further reinforced by gender considerations: in a pivotal family of size $2K$ the K'th child is a boy, and thus will get more investment (and take more investment away from children $K+1...2K-2$) than the K'th child in a pivotal family of size $2K-2$ (who is a girl). Also, the comparison of $\delta^{K-1}\hat{x}_K$ versus $\delta^{2K-2}x_{m_{2K-1}} + \delta^{2K-1}x_{m_{2K}}$ favors the left hand side of Equation 8, because investments are decreasing in a child's rank. Thus, if there is any quality-quantity tradeoff (Becker and Lewis 1973) investment decisions reinforce the tendency of decreased family sizes to lead to increased equality.

Proposition 4 *If $\hat{x}_{m_k} \geq x_{m_k}, \forall k \in \{K+1, ..., 2K-2\}$ with a strict inequality for at least one k, then the set $\{\delta | e(K-1) > e(K)\}$ is a strict superset of the values of δ for which equality increases as fertility decreases in the baseline model.*

It is important to note that this result does not require that allocation of resources between boys and girls become more equitable as family size decreases, just that the resources allocated to girls don't decrease. Empirical work by Yount et al (2014) shows that fertility declines are correlated with better outcomes for both boys and girls, though sometimes the gains for boys are larger than for girls.[5] Studies that use twin births as an instrument find null effects in Norway (Black, Devereux, and Salvanes 2005), Israel (Angrist, Lavy, and Schlosser 2010) and Brazil (Marteleto and de Souza 2013), but find a quantity-quality tradeoff in India (Rosenzweig and Wolpin 1980) and China (Rosenzweig and Zhang 2009). For a summary of the literature on quantity-quality tradeoffs see Doepke (2015).

Discussion

This paper formalizes the intuition that decreases in family size lead men in patriarchal societies to support increased equality of economic opportunity for women, due to concerns over opportunities for their daughters. The main result is that this effect occurs in societies with a "winners-take-most" approach to the relative importance of different children. In such societies, a patriarch is more inclined to support economic opportunities for his daughters when there are fewer boys above them in the intra-family hierarchy. In contrast, in societies that are intra-family egalitarian, decreases in family size actually lead to decreased economic opportunity for women, because patriarchs have fewer daughters who are negatively affected by lack of opportunities. These results continue to hold basically unchanged if each family's size is chosen endogenously via a stopping rule.

I also analyze two factors that, while not necessary for the main result, make it more likely that reduced fertility will lead to increases in equality of opportunity. The first of these factors is a patriarch's concern that competition from women will harm the labor market prospects or status of

[5]There are many other correlational studies: Bhat (2002) on India; Buchmann (2000) on Kenya; DeGraff, Bilisborros, and Herrin (1996) and Gipson and Hindin (2015) on the Philippines; Knodel and Wongsith (1991) on Thailand; Lachaud et al (2014) on Burkina Faso; Parish and Willis (1993) and Yu and Su (2006) on Taiwan; Sudha (1993) on Malaysia; Tsui and Rich (2002) and Ye and Wu (2011) on China; and a review article by Steelman et al (2002) that finds mixed effects

his sons. The second factor is variation in the allocation of resources within households, as long as there is a weak quality-quantity tradeoff for children.

To test this argument empirically could be challenging, due to the difficulty of finding an instrument for fertility. Some natural candidates, e.g., war or economic depression, would obviously fail to satisfy the exclusion restriction. Similarly, availability of contraception, which could clearly affect fertility, may be affected by omitted variables that also directly affect government policies for equality of opportunity.

Appendix

Proof of Observation 1 Let $\bar{p} \equiv \max\left\{p | (p-1,p) \notin \hat{S}(e) \text{ or } (p,p-1) \notin \hat{S}(e)\right\}$ be the largest number of girls and boys that is ever reached by any family with an equal gender split in the stopping problem. There are three possibilities.

Case 1: $(\bar{p},\bar{p}) \notin \hat{S}(e)$. In this case, after having $\bar{p}$ girls and $\bar{p}$ boys, a family will have exactly one more child and then stop, regardless of the child's gender. Half of the families have $b_i \geq \bar{p}+1$ boys and $g_i \leq \bar{p}$ girls, and of these, the highest e_i is for patriarchs with $\bar{p}+1$ boys and $\bar{p}$ girls. Half of the families have $b_i \leq \bar{p}$ boys and $g_i \geq \bar{p}+1$ girls, and of these, the lowest e_i is for patriarchs with exactly $\bar{p}$ boys and $\bar{p}+1$ girls. [[Note: In this draft, I'm not proving statements about the proportion of families with different compositions, e.g., that half of the families will have $b_i \geq \bar{p}+1$ boys and $g_i \leq \bar{p}$ girls. But it seems pretty obvious that such statements are true.]]

Case 2: $(\bar{p},\bar{p}) \in \hat{S}(e)$. In this case, a family with $\bar{p}$ girls and $\bar{p}$ boys will not have an additional child. This case can be divided into two subcases, based on the behavior of families with $\bar{p}$ children of one gender and $\bar{p}-1$ children of the other gender.

Subcase 2a: $(\bar{p},\bar{p}-1) \notin \hat{S}(e)$ and $(\bar{p}-1,\bar{p}) \notin \hat{S}(e)$. In this subcase, any family with $\bar{p}$ children of one gender and $\bar{p}-1$ children of the other gender will have another child. Strictly less than half of the families have $b_i \geq \bar{p}+1$ boys and $g_i \leq \bar{p}$ girls and strictly less than half of families have $b_i \leq \bar{p}$ boys and $g_i \geq \bar{p}+1$ girls. The median e_i is for a patriarch with exactly $\bar{p}$ children of each gender, as in the baseline model.

Subcase 2b: $(\bar{p}, \bar{p}-1) \notin \hat{S}(e)$ and $(\bar{p}-1, \bar{p}) \in \hat{S}(e)$. In this subcase, a family with $\bar{p}$ boys and $\bar{p}-1$ girls will have an additional child, but one with $\bar{p}-1$ boys and $\bar{p}$ girls won't.[6] Half of the families have $b_i \geq \bar{p}$ boys and $g_i \leq \bar{p}-1$ girls and of these the highest e_i is for patriarchs with $\bar{p}$ boys and $\bar{p}-1$ girls. Half of the families have $b_i \leq \bar{p}$ boys and $g_i \geq \bar{p}$ girls and of these, the lowest e_i is for patriarchs with exactly $\bar{p}$ boys and $\bar{p}$ girls.

To have a family with (b, g) and $b = g = n$ be pivotal requires two conditions, from Subcase 2a. First, both $(n, n-1) \notin \hat{S}(e)$ and $(n-1, n) \notin \hat{S}(e)$. The latter constraint is the binding one because the incentives to have another child are higher with fewer boys. From Equation 6, satisfying this constraint requires

$$\delta^{2n-1}\gamma e + \frac{1}{2}\delta^{n-1}(1-\gamma e) > c.$$

The second requirement for (b, g) and $b = g = n$ to be pivotal is that $(n, n) \in \hat{S}(e)$, i.e., from Equation 6

$$\delta^{2n}\gamma e + \frac{1}{2}\delta^{n}(1-\gamma e) < c.$$

Thus the range of c for which (b, g) with $b = g = n$ is the pivotal family composition is

$$\delta^{2n}\gamma e + \frac{1}{2}\delta^{n}(1-\gamma e) < c < \delta^{2n-1}\gamma e + \frac{1}{2}\delta^{n-1}(1-\gamma e). \tag{9}$$

To have a family with (b, g) and $b - 1 = g = n$ be pivotal as in Case 1 requires two conditions. First, $(n, n) \notin \hat{S}(e)$, which, from Equation 6, requires

$$\delta^{2n}\gamma e + \frac{1}{2}\delta^{n}(1-\gamma e) > c.$$

[6]It is impossible to have $(\bar{p}, \bar{p}-1) \in \hat{S}(e)$ and $(\bar{p}-1, \bar{p}) \notin \hat{S}(e)$ because the marginal benefit of another child in Equation 6 is higher with more daughters, holding family size constant.

The second requirement for (b, g) where $b - 1 = g = n$ to be pivotal as in Case 1 is that $(n, n + 1) \in \hat{S}(e)$, which, from Equation 6, requires

$$\delta^{2n+1}\gamma e + \frac{1}{2}\delta^{n}(1 - \gamma e) < c.$$

Thus the range of c for which (b, g) with $b = g + 1 = n$ is pivotal as in Case 1 is

$$\delta^{2n+1}\gamma e + \frac{1}{2}\delta^{n}(1 - \gamma e) < c < \delta^{2n}\gamma e + \frac{1}{2}\delta^{n}(1 - \gamma e). \tag{10}$$

A family with (b, g) and $b - 1 = g = n$ can also be pivotal as in Subcase 2b. This requires two conditions. First, $(n + 1, n) \in \hat{S}(e)$ which, from Equation 6, requires

$$\delta^{2n+1}\gamma e + \frac{1}{2}\delta^{n+1}(1 - \gamma e) < c.$$

The second requirement for (b, g) where $b - 1 = g = n$ to be pivotal as in Subcase 2b is that $(n, n + 1) \notin \hat{S}(e)$, which, from Equation 6, requires

$$\delta^{2n+1}\gamma e + \frac{1}{2}\delta^{n}(1 - \gamma e) > c.$$

Thus the range of c for which (b, g) with $b - 1 = g = n$ is pivotal as in Subcase 2b is

$$\delta^{2n+1}\gamma e + \frac{1}{2}\delta^{n+1}(1 - \gamma e) < c < \delta^{2n+1}\gamma e + \frac{1}{2}\delta^{n}(1 - \gamma e). \tag{11}$$

Combining Equations 10 and 11 we get the full range of c for which $b - 1 = g = n$ is pivotal (in either Case 1 or Subcase 2b):

$$\delta^{2n+1}\gamma e + \frac{1}{2}\delta^{n+1}(1 - \gamma e) < c < \delta^{2n}\gamma e + \frac{1}{2}\delta^{n}(1 - \gamma e). \tag{12}$$

Observation 1 combines the results in Equations 9 and 12, with $c_{n,n}(e) = \delta^{2n}\gamma e + \frac{1}{2}\delta^{n}(1 - \gamma e)$ and $c_{n+1,n}(e) = \delta^{2n+1}\gamma e + \frac{1}{2}\delta^{n+1}(1 - \gamma e)$.

Proof of Proposition 3 If $b = g$ the proof identical to the proof for Proposition 1, setting $b = g = K$. If $b = g + 1$, the argument is virtually identical. As in Equation 2, the first order condition for $e(b, g)$ is

$$D'(e(b,g)) = -\sum_{k=b+1}^{2b-1} \delta^{k-1}\gamma$$

and for $e(b-1, g-1)$ it is

$$D'(e(b-1,g-1)) = -\sum_{k=b}^{2b-3} \delta^{k-1}\gamma.$$

Thus, $e(b-1, g-1) > e(b, g)$ iff

$$\begin{aligned} \sum_{k=b}^{2b-3} \delta^{k-1}\gamma &> \sum_{k=b+1}^{2b-1} \delta^{k-1}\gamma \\ \delta^{b-1} &> \delta^{2b-2} + \delta^{2b-3} \\ 1 &> \delta^{b-1}(1+\delta). \end{aligned}$$

Proof of Proposition 4 First note that $\hat{x}_{m_k} \geq x_{m_k}, \forall k \in \{K+1, ..., 2K-2\}$ implies that

$$\sum_{k=K+1}^{2K-2} \delta^{k-1}\hat{x}_{m_k} > \sum_{k=K+1}^{2K-2} \delta^{k-1}x_{m_k}.$$

Also, note that because $\hat{x}_{m_k} \geq x_{m_k}, \forall k \in \{K+1, ..., 2K-2\}$ and investment is decreasing in rank, $\hat{x}_{m_K} \geq x_{m_K} > x_{m_{2K-2}} > x_{m_{2K-1}} > x_{m_{2K}}$. Thus the set of δ for which

$$\delta^{K-1}\hat{x}_{m_K} + \sum_{k=K+1}^{2K-2} \delta^{k-1}\hat{x}_{m_k} > \sum_{k=K+1}^{2K-2} \delta^{k-1}x_{m_k} + \delta^{2K-2}x_{m_{2K-1}} + \delta^{2K-1}x_{m_{2K}}$$

is a strict superset of the set of δ for which

$$\delta^{K-1} > \delta^{2K-2} + \delta^{2K-1}$$

which is the condition in Equation 3 for the baseline model.

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