

Winning by Default: Why is There So Little Competition in Government Procurement?

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Abstract

In government procurement auctions, eligibility requirements are often imposed and, perhaps not surprisingly, contracts generally have a small number of participating bidders. Using the federal government procurement data, we study the determinants of such restrictions of competition, and find evidence that less competition is correlated with various measures of political connection and corruption. To understand the effects of the restrictions of competition on the total cost of government procurement, we develop, identify, and estimate a principal-agent model in which the government selects a contractor to undertake a project. We consider three reasons why restricting entry could be beneficial to the government: by decreasing bid processing and solicitation costs, by increasing the chance of selecting a favored contractor and consequently reaping benefits from the favored contractor, and by decreasing the expected amount of payment to the winning contractor. When the participation is costly and bidders are heterogeneous, the expected amount of payment to the winning contractor may decrease by restricting entry to a certain set of contractors. Using our estimates, we decompose the effects of these three sources of entry restrictions, and quantify the effects of the eligibility restrictions on the total cost of procurement.

1 Introduction

In recent ten years, the market for the United States federal government procurement is worth over \$460 billion annually, which constitutes about 18% of the yearly federal government spending. Despite its vast size, the extent of competition for a procurement contract is not

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very intense. Among the procurement contracts awarded during 2004–2012, about 43% of them were awarded under either limited or no competition. Even when a contract was open to full competition, attracting only one bidder for the contract was not uncommon. In this paper, we develop, identify, and estimate a model of procurement to better understand this low extent of competition observed in the data.

There are three important institutional features of federal government procurement that have received relatively little attention from the literature, both theoretically and empirically. First, the number of bids is relatively small; in fact, winning a contract by default is not uncommon in federal government procurement. Second, the government often chooses to impose eligibility restrictions. Although full and open competition is encouraged, restricting entry is allowed under various circumstances by regulation. Third, a sealed-bid auction is not always a dominant procedure to choose a contractor, depending on the nature of the products or services to be procured. According to the Federal Acquisition Regulation, a sealed-bid auction is not considered appropriate to use if the award will be made on the basis of factors other than price; if it is necessary to conduct discussions with the responding contractors about their bids; or if there is not a reasonable expectation of receiving more than one sealed bid. An alternative solicitation procedure is *competitive proposal evaluation*, through which the proposals submitted by contractors are evaluated, negotiated, and selected. Under this procedure, the contract terms and the winner are simultaneously determined.

In this paper, we construct a principal-agent framework that incorporates these three features. In our model, the principal (the procurer) chooses the extent of competition, i.e., the eligibility conditions and the expected number of bidders, and offers a menu of contracts to the participating agents (the contractors) with a hidden type (cost). A contract chosen by a contractor is interpreted as a bid, which reflects the competitive proposal evaluation procedure. This procedure can be more profitable to the procurer than a sealed-bid auction when the number of bidders is very small.

The procurer may choose to decrease the extent of competition for three reasons. First, bid processing and solicitation costs may increase as the number of bidders increases. These costs include the time cost of waiting to receive more bids and the cost of reading the proposals and assessing various attributes of the contractors.

Second, decreasing competition may increase the chance that a favored contractor will win the contract. If the procurer receives positive benefits from hiring a favored contractor and there are regulations that prohibit the procurer from awarding a contract to an inefficient, favored contractor over an efficient, unfavored contractor, the procurer may want to decrease the extent of competition. Here, the benefits from hiring a favored contractor are not necessarily related to corruption; for example, a contractor may be favored by the

procurer because of its quality of work, which cannot be easily measured or quantified.

Third, a larger number of bidders or an unrestricted competition does not guarantee a lower expected amount of payment to the winning contractor. The paradox of the possible negative relationship between the procurer’s revenue and the extent of competition in independent private auctions has been studied by Li and Zheng (2009), Krasnokutskaya and Seim (2011), and Athey, Coey and Levin (2013), for example. In these papers, this paradox is solved by allowing the contractors to choose to enter at a cost. They show that accounting for participation is important in assessing policies that restrict competition. In our paper, the probability of hiring an efficient contractor is allowed to increase or decrease as the number of participants increases.

Using the federal procurement contracts awarded during 2004–2012, we first document the determinants of the extent of competition. For each procurement contract, we calculate the ratio of the conviction cases of public office abuse to the size of public employees in the state where the contract is to be performed. We find that a higher rate of corruption is associated with a higher frequency of having some eligibility restrictions. We also find that when a winning bidder is represented by a member in the House Appropriations Committee, it is more likely to win the contract among a smaller number of bidders. The extent of this correlation is found to be much larger when the winning bidder is represented by top five members from the majority party in the House Appropriations Committee. These findings suggest that political corruptions or connections partially explain the extent of competition for procurement contracts.

To further investigate the reasons why there is relatively little competition in the federal procurement contracts, we nonparametrically estimate the model following our identification argument. Using the estimates, we conduct counter-factual analyses to decompose the effects of the three sources of entry restrictions. We also quantify the effects of the eligibility restrictions on the total cost of procurement.

The rest of the paper is organized as follows. Section 2 describes the solicitation and source selection procedures in the federal procurement, which is modeled in Section 3. In Section 4, we introduce our data and present descriptive analyses. Then we discuss identification of the model in Section 5. We closely follow the identification argument to nonparametrically estimate our model, and we show the estimation results in Section 6. Section 7 concludes.

1.1 Related Literature

Our paper is related to the large literature on procurement and auctions. One strand of the literature explains why less competition does not necessarily lower the payoff of the auctioneer in independent private value auctions. Li and Zheng (2009) show that when the number of bidders is endogenously determined, bidders equilibrium bidding behavior can become less aggressive as the number of potential bidders increases. It is because when the number of potential bidders increases, the bidders equilibrium entry probability decreases, i.e., the expected number of actual bidders decreases. Krasnokutskaya and Seim (2011) study a bid preference program, and Athey, Coey and Levin (2013) compare a set-asides program and the bid subsidy program. Both papers show the importance of allowing endogenous entry when assessing restrictive competition policies. In our paper, the procurer optimally chooses the extent of competition, considering bid processing and solicitation costs, favoritism, and endogenous participation of bidders. Furthermore, the procurer may give a precedence to a favored group of bidders, which provides incentives to nonfavored firms to bid more aggressively against the favored bidders. This may decrease the cost of procurement, as pointed out by McAfee and McMillan (1989).

Another strand of the literature studies nonstandard contractor selection procedures, such as scoring auctions (Asker and Cantillon (2010)) or multi-attribute auctions (Krasnokutskaya, Song and Tang (2013)), where the price is not the only factor in selecting a contractor. Our paper is closely related to Laffont and Tirole (1987), where a bid consists of an expected price and of a coefficient of sharing of cost overruns. We extend their model by allowing the procurer to choose the optimal extent of competition. [mention McAfee and McMillan (1986) and McAfee and McMillan (1987)]

Lewis and Bajari (2014) and Bajari, Houghton and Tadelis (2014) are related to our paper in that they study the payment adjustments after the winning contractor is chosen and the project initiates. We distinguish the payment adjustments into two categories: those incurring due to the cost changes related to the unknown type of the contractors and the rest. Lewis and Bajari (2014) study the former type of the payment adjustments, and Bajari, Houghton and Tadelis (2014) study the latter. We consider both types of the payment adjustment, and this is possible because we observe the reasons for contract modifications.

Our paper also belongs to a literature on the identification of a principal-agent model with adverse selection, for example, Perrigne and Vuong (2011). In their paper, an implicit identifying assumption is that the optimal contracts are linear in costs. Their theoretical model provides the payoffs of the principal and the agent, but it does not provide a particular contractual form that an optimal contract must follow. In our identification result, we do not impose any functional form assumptions on the optimal contracts.

Lastly, our paper is related to the political economy literature on how the federal government funds are allocated to the state or local governments or the private entities. For example, Knight (2005) show that members in the transportation committee secure higher project spending than do members from other districts. De Figueiredo and Silverman (2006) find that universities represented by a House or Senate Appropriations Committee member receive benefits regarding earmarks. In our paper, we document that the contractors represented by such member tend to face less competition when bidding for a procurement project.

2 The Solicitation and Source Selection Procedures

We provide a brief description of the U.S. federal government’s process of soliciting and awarding a procurement project, emphasizing key features which motivate our modelling approach. As the procurement process begins, contracting officers decide the extent and method by which the contract will be competed. Although they are expected to promote full and open competition, they have broad discretion in choosing the extent of competition. The regulations allow contracting officers to exclude some sources and to award sole-source contracts under certain conditions. For example, the Clinger-Cohen Act of 1996 gives contracting officers the authority to eliminate bidders from consideration if the contracting officer determines that the number of bidders would otherwise be considered are too many to evaluate efficiently.

Table 1 shows the summary statistics on the extent of competition and the solicitation procedure for all definitive contracts and purchase orders of an expected size of \$300,000 or more during the period of FY 2000-2014. This size threshold is chosen because the contracts of an anticipated size greater than \$300,000 are not normally expected to be reserved exclusively for small business concerns.

There are three key trends in the data on the solicitation and selection procedures that inform our modelling choices. First, for more than half of the projects in the data, the number of bids is one. Out of the total payment to the contractors in this sample (\$2.7 trillion), \$1.4 trillion of the payments were made to the winners by default.

Second, certain contractors were not eligible to participate in the competition for 234,863 projects in total, about 73% of the projects in this sample. Among them, 56,238 projects were not even competed due to various statutes and regulations, and the rest were competed after some contractors are excluded from participation. Some projects, often of a smaller size, are set aside for small businesses, which are denoted as “Set-asides” in the table. Any other type of exclusion of sources are denoted as “Exclusive.” The Federal Acquisition Regulation

Table 1: The Extent of Competition and the Solicitation Procedure

	Obs.	Mean Payment (\$K)	Median Payment (\$K)	Mean # of Bids	Median # of Bids
Sealed-Bid	23,355	2,953.6	956.5	7.2	4
<i>Open</i>	15,148	3379.6	986.8	6.7	3
<i>Exclusive</i>	2,817	2,513.5	10.7	7.2	7
<i>Set-asides</i>	5,390	1,986.9	7.0	7.2	5
Not Sealed-Bid	238,617	10,452.6	680.5	13.5	2
<i>Open</i>	124,325	11,475.4	702.5	10.4	2
<i>Exclusive</i>	87,489	11,334.0	640.8	15.2	1
<i>Set-asides</i>	26,803	2,831.2	737.3	22.3	3
Not Competed	56,238	2,840.4	749.6	1	1
Total	318,210	8,556.9	707.9	10.8	1

* Note: The amount of payment is CPI-adjusted, where CPI of December 2010 is 100. Peer reviewed basic research contracts are excluded in this table.

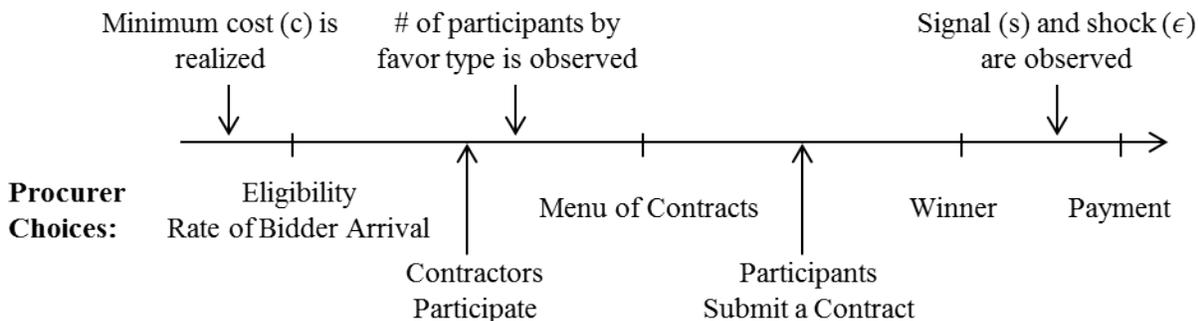
specifies the list of circumstances under which such limitation of competition is allowed.

Third, the seal-bid auction is not a dominant solicitation procedure. The solicitation procedure for about 7% of the projects (23,355 projects) in this sample is a sealed-bid auction. The most prevalent solicitation procedure is the competitive proposal evaluation, through which the proposals submitted by contractors will be evaluated, negotiated, and selected. After the request for proposals is posted, the qualified contractors can submit their proposals, which will be reviewed in detail to determine which proposals are within a competitive range. Discussions and negotiations may then be carried out with the contractors within the competitive range, and the contractor will be selected whose proposal is found to be most advantageous to the procuring agency. During the discussions and negotiations, the contract type and prices are considered to be closely related and are considered together.

3 Model

This section lays out a procurement model in which a procurer selects a contractor from multiple agents to undertake a project. The procurer chooses the extent of competition by restricting the group of agents who are eligible to participate and deciding the amount of search efforts to receive enough bids. We characterize the equilibrium extent of competition, selection, and contract for this model, along with the probability distributions that determine the selection of a contractor for undertaking the project, the resources that the contractor devotes to the project, the time to project completion, and the payouts to the contractor. A simple timeline is depicted in Figure 1.

Figure 1: Timeline of the Procurement Process in the Model



To begin with, we consider the optimal menu of contracts when only one contractor participates. Second, we consider the optimal selection mechanism when multiple contractors show interest in doing the procurement project. Once the agents participate in the procurement process, the procurer announces a menu of contracts. Then the participating agents simultaneously submit proposals, each one choosing an item from the menu if the expected rent from doing so is nonnegative. The procurer selects a contractor, who undertakes the project according to his submission. Lastly, we discuss the optimal extent of competition. The proofs for theorems and lemmas are in Appendix.

3.1 Optimal Menu of Contracts with Single Bidder

Consider a procurement project and suppose only one contractor shows interest in doing the project. The cost of completing the project depends on the minimum cost (c), the extent of inefficiency of the agent (β), and the ex-post cost changes due to stochastic realizations of demand or supply shocks (ϵ):

$$c + \beta + \epsilon.$$

The realization of c and ϵ is observed by both the procurer and the agent at the same time: c is observed before the project is let and ϵ is observed after the project is initiated. We assume that c and ϵ are distributed independently of β , and we denote the distribution of c as $G(\cdot)$. We assume that the distribution has a bounded support, $[c^L, c^U]$. In completing the project, a signal, denoted by s , is revealed to both the procurer and the contractor. The signal is drawn from cumulative density function $F(s|\beta)$. The procurer cannot directly observe the cost of completing the project or β , but does observe s . We assume that the support of s is invariant to β .

The objective of the procurer, upon the participation of one contractor, is to minimize the payment to the contractor in completing the project. To do so, the procurer may present

a menu of contracts to the contractor to elicit the type of the contractor. A typical contract has two components, a base payment, p , and a plan of ex-post payment adjustments, $r(\cdot)$, which are contingent on the realization of the signal and the cost shock ϵ . Given the realized value of s , and ϵ the payoff to the agent under the contract is

$$U(s, \epsilon; p, r(\cdot), \beta) = p - (c + \beta) + \psi(r(s, \epsilon) - \epsilon),$$

where $\psi(\cdot)$ is a continuous function, with $\psi(0) = 0$, $\psi'(0) = 1$, $\psi' > 0$, and $\psi'' < 0$. Note that $p - (c + \beta)$ is the fixed part of the payoff and $r(s, \epsilon) - \epsilon$ is the variable part of the payoff. Due to liquidity concerns, the variable part of the payoff is discounted, which is represented by $\psi(\cdot)$. However, the procurer does not have liquidity concerns, and as a result, the cost to the procurer is $p + r(s, \epsilon)$.

We solve for the Bayesian equilibrium that minimizes the expected payment to the sole bidder conditional on completing the project. The optimal contract would be easy to implement if the procurer directly observed the efficiency of the participating agents. If an agent with β is selected as contractor, the procurer demands a project with minimum cost c in return for $c + \beta + \epsilon$. Consequently the contractor does not receive any rent from the procurement award and costly signaling is redundant. The full information solution may be infeasible if type is unobserved because efficient agents have an incentive to masquerade as inefficient. We derive the menu of optimal contracts under this scenario.

Note that first the procurer does not have liquidity concerns while the agents have them; and second, the cost shock, represented by ϵ , is independent of β . Therefore, the procurer minimizes her expected payment by fully insuring the contractors against the cost shock. This is represented by the following lemma.

Lemma 1 *Consider any optimal menu of n contracts, $\{p_1, r_1(s, \epsilon); \dots; p_n, r_n(s, \epsilon)\}$. For all $j = 1, \dots, n$, there exists $q_j(\cdot)$ such that the following equation holds for all (s, ϵ) :*

$$r_j(s, \epsilon) \equiv \epsilon + q_j(s).$$

Given this lemma, we focus on solving the optimal menu of the base payments (p) and the ex-post payment adjustments conditional on the signal ($q(s)$). Here, we assume that there are two values of efficiency, $\underline{\beta} = 0$ and $\bar{\beta} > 0$. We assume that the procurer's subjective probability that the sole bidder is efficient is π . We show that the procurer minimizes the expected payment by offering one fixed-price contract or a menu of two contracts. The menu consists of two contracts: (i) \underline{p} and (ii) $(\bar{p}, \bar{q}(s))$. In equilibrium, the efficient agent selects the former contract, and the inefficient one chooses the latter contract.

Theorem 1 *The equilibrium menu of contracts is either a single fixed-price contract $c + \bar{\beta}$ or a menu of two contracts comprising a fixed-price contract, denoted by \underline{p} , that the efficient agent selects, and a variable-price contract, denoted by $(\bar{p}, \bar{q}(s))$, that the inefficient agent accepts.*

If both fixed-price and variable-price contracts are offered, the fixed-price contracts are only chosen by efficient agents. For suppose to the contrary that an inefficient agent chose the fixed-price contract: then the fixed-price would be at least $c + \bar{\beta}$, and the efficient agent could extract all the rent of his efficiency, $\bar{\beta}$, by selecting the same contract. The procurer could reduce her expected costs of procurement below the pooling equilibrium outcome if there is a set of informative signals, meaning $f(s|\underline{\beta}) \neq f(s|\bar{\beta})$, that occurs with strictly positive probability. In that case the principal could offer two contracts, a variable-price contract that is increasing the likelihood ratio $f(s|\underline{\beta}) / f(s|\bar{\beta})$ that leaves the inefficient agent indifferent between undertaking the project versus not participating, as well as a fixed-price contract, with payment between c and $c + \bar{\beta}$, which only the efficient agent would take. Thus the principal balances the losses associated with the liquidity premium paid to the inefficient agent $\mathbb{E}[\psi(q)|\bar{\beta}]$ against the constraint of offering a sufficiently high fixed-price contract to attract the efficient agent.

Now, we characterize the equilibrium menu of two contracts. The objective of the procurer is to minimize the expected payment to the winning contractor:

$$\pi \underline{p} + (1 - \pi) (\bar{p} + \mathbb{E}(\bar{q}|\bar{\beta})).$$

To induce a type β to choose $(p, q(\cdot))$, the menu of contracts must satisfy the individual rationality constraint,

$$p + \mathbb{E}(\psi(q)|\beta) - (c + \beta) \geq 0,$$

and the incentive compatibility constraint,

$$p + \mathbb{E}(\psi(q)|\beta) \geq p' + \mathbb{E}(\psi(q')|\beta),$$

for any other contract $(p', q'(\cdot))$ in the menu. We show that at the optimum, the individual rationality constraint for the inefficient type and the incentive compatibility constraint for the efficient type always bind. The other two constraints do not bind at the optimum. The characterization of the optimal menu of contracts are below.

Theorem 2 *If the optimal menu of contracts contains two contracts, one fixed-price contract*

\underline{p} and one variable-price contract $\{\bar{p}, \bar{q}\}$, then the fixed-price contract is:

$$\underline{p} = c + \bar{\beta} - \int \psi(\bar{q}(s)) [f(s|\bar{\beta}) - f(s|\underline{\beta})] ds. \quad (1)$$

The variable-price contract consists of the base payment:

$$\bar{p} = c + \bar{\beta} - \int \psi(\bar{q}(s)) f(s|\bar{\beta}) ds, \quad (2)$$

and the ex-post payment adjustment schedule. The adjustment schedule is two-tiered. If $f(s|\underline{\beta}) / f(s|\bar{\beta}) \geq 1/\pi$, $\bar{q}(s) = M$, where $M < 0$ is the maximal penalty that the government can legally impose on contractors. If $f(s|\underline{\beta}) / f(s|\bar{\beta}) < 1/\pi$, then the ex-post payment adjustment schedule satisfies the following equation:

$$\psi'(\bar{q}(s)) \left[1 - \pi \frac{f(s|\underline{\beta})}{f(s|\bar{\beta})} \right] = 1 - \pi. \quad (3)$$

In the following theorem, we specify the conditions under which offering a menu of two contracts is optimal for the procurer.

Theorem 3 *If there exists a set of signals, S , such that*

$$\Pr(s \in S | \underline{\beta}) \equiv \gamma_1 < \Pr(s \in S | \bar{\beta}) \equiv \gamma_2,$$

then the equilibrium is separating. Otherwise, the equilibrium is pooling.

3.2 Optimal Menu of Contracts with Multiple Bidders

When multiple contractors show interest in doing the project, the procurer must choose one contractor. Some of these participating contractors may be favored by the procurer for various reasons. We consider a second dimension of heterogeneity among the contractors, in addition to their efficiency: whether or not a contractor is favored or not. This favoritism is reflected in the payoff to the procurer, in that when a favored agent wins the project, the procurer receives monetary benefit $\delta > 0$. In the following, we consider three cases of multiple participants: first, when all participants are of the same type when it comes to favoritism; and second, when some are favored and some are not. We assume that when the bidders submit their contract or bid, they do not know the efficiency of other participants and have the same belief on the distribution of other participants' efficiency as the procurer's. They are assumed to know whether or not each of other participants are favored by the procurer.

Multiple Homogeneous Bidders Consider n contractors who are all favored by the procurer. Similarly in the one-participant case, it is optimal to offer either one fixed-price contract or a menu of two contracts, $\{\underline{p}, (\bar{p}, \bar{q}(\cdot))\}$, and to select a agent with the highest efficiency among the participating bidders. When one fixed-price contract is offered, the procurer is indifferent and chooses a random participant. When two contracts are offered, it is optimal to select a participant that accepts the fixed-price contract, if there's one. If no one accepts the fixed-price contract, then the procurer randomly picks one participant. Given this bidder-selection rule, the expected payment to a winning contractor when two contracts are offered is:

$$\Pr(\text{fixed}|n, 0)\underline{p} + (1 - \Pr(\text{fixed}|n, 0)) [\bar{p} + \mathbb{E}(\bar{q}(s)|\bar{\beta})],$$

where the probability that the winning contractor is efficient (i.e., the winning contract is fixed-price), denoted by $\Pr(\text{fixed}|n, 0)$, is

$$\Pr(\text{fixed}|n, 0) = 1 - (1 - \pi_f(n))^n, \quad (4)$$

where $\pi_f(n)$ is the ratio of efficient agents among n favored participants. To induce an efficient contractor to choose the fixed-price contract, \underline{p} , the following individual rational constraint and the incentive compatibility constraint must be satisfied.

$$\underline{p} - (c + \underline{\beta}) \geq 0,$$

$$\underline{\phi}(n)\{\underline{p} - (c + \underline{\beta})\} \geq \bar{\phi}(n)\{\bar{p} + \mathbb{E}(\psi(q)|\bar{\beta}) - (c + \underline{\beta})\},$$

where $\underline{\phi}(n)$ is the subjective probability that a contractor will win a project if he submits the fixed-price contract given that other participating contractors follow the equilibrium strategy, and $\bar{\phi}(n)$ is the winning probability if he submits the variable-price contract:

$$\underline{\phi}(n) = \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{\pi_f(n)^k (1 - \pi_f(n))^{n-1-k}}{k+1},$$

$$\bar{\phi}(n) = \frac{(1 - \pi_f(n))^{n-1}}{n}.$$

Note that $\underline{\phi}(n) > \bar{\phi}(n)$ for any n and $\bar{\phi}(n)/\underline{\phi}(n)$ is decreasing in n , as long as $\pi_f(n) < 1$. Similarly, the following two constraints for inefficient contractors must be satisfied:

$$\bar{p} + \mathbb{E}(\psi(q)|\bar{\beta}) - (c + \bar{\beta}) \geq 0,$$

$$\bar{\phi}(n)\{\bar{p} + \mathbb{E}(\psi(q)|\bar{\beta}) - (c + \bar{\beta})\} \geq \underline{\phi}(n)\{\underline{p} - (c + \bar{\beta})\}.$$

Following the same logic in Theorem 2, the individual rational constraint for the inefficient contractor and the incentive compatibility constraint for the efficient contractor are binding at the optimum. This characterizes the optimal menu of contract in the following theorem.

Theorem 4 *Suppose there are n favored participants for a project. If the optimal menu of contracts contains two contracts, one fixed-price contract \underline{p} and one variable-price contract $\{\bar{p}, \bar{q}\}$, then the fixed-price contract is:*

$$\underline{p} = c + \underline{\beta} + \frac{\bar{\phi}(n)}{\underline{\phi}(n)} \left[\bar{\beta} - \underline{\beta} - \int \psi(\bar{q}(s)) [f(s|\bar{\beta}) - f(s|\underline{\beta})] ds \right]. \quad (5)$$

The variable-price contract consists of the base payment:

$$\bar{p} = c + \bar{\beta} - \int \psi(\bar{q}(s)) f(s|\bar{\beta}) ds, \quad (6)$$

and the ex-post payment adjustment schedule. The adjustment schedule is two-tiered. If s is such that

$$\frac{f(s|\underline{\beta})}{f(s|\bar{\beta})} < 1 + \frac{n(1 - \pi_f(n))\underline{\phi}(n)}{1 - n(1 - \pi(n))\bar{\phi}_f(n)},$$

then the ex-post payment adjustment schedule satisfies the following equation:

$$\psi'(\bar{q}(s)) \left(1 + \frac{1 - n(1 - \pi_f(n))\bar{\phi}(n)}{n(1 - \pi(n))\underline{\phi}(n)} \left[1 - \frac{f(s|\underline{\beta})}{f(s|\bar{\beta})} \right] \right) = 1. \quad (7)$$

Otherwise, $\bar{q}(s) = M$, where $M < 0$ is the maximal penalty that the government can legally impose on contractors.

Note that when $n = 1$, $\bar{\phi}(1) = \underline{\phi}(1)$; therefore, equations (5) and (7) in Theorem 4 are the same as equations (1) and (3) in the single-participant case, Theorem 2.

When all participants are unfavored by the procurer, the optimal menu of contracts and the bidder selection rule are the same as the case when all participants are favored, except that $\pi_f(n)$ is replaced by $\pi_u(n)$, where $\pi_u(n)$ is the ratio of efficient agents among n unfavored participants.

Multiple Heterogeneous Bidders The procurer receives a nonnegative benefit δ from selecting a favored agent. We assume that she is not allowed to choose a favored, inefficient

agent if a non-favored, efficient agent participates in the competition.¹ Therefore, her selection rule is lexicographic: the first criterion is efficiency and the second criterion is favoritism. As a result, if at least one favored agent participates, inefficient and unfavored agents never win a project.

Because the ratio of efficient agents may be different for the group of the favored agents compared to the group of the unfavored agents, it is optimal for the procurer to offer different menus of contracts based on whether or not an agent is favored. Note that when at least one participant is favored, the procurer does not have to satisfy the participation condition for an inefficient and unfavored agent. Therefore, the procurer will offer a fixed contract with base payment c to unfavored agents. For the favored agents, a menu of two contracts can be offered as long as the signal is informative as discussed in Theorem 3. The optimal menu of contracts can be characterized similarly as in Theorem 4 because this menu is offered to favored contractors only. The following theorem summarizes the optimal contracts.

Theorem 5 *Suppose there are $n_f \geq 1$ favored participants and $n_u \geq 1$ unfavored participants for a project. To the unfavored contractors, a fixed-price contract, c , is offered. To the favored contractors, either a fixed-price contract, $c + \bar{\beta}$ or a menu of two contracts, consisting of one fixed-price contract and one variable-price contract, is offered.*

3.3 Optimal Extent of Competition

The procurer chooses the extent of the eligibility to participate in the procurement and the amount of bid solicitation efforts. First, she may limit the eligibility of unfavored agents for participation in the procurement process. This exclusive competition guarantees that only favored agents can win the project, which provides additional monetary benefit to the procurer, $\delta > 0$.

Second, the procurer also determines how many contractors to solicit bids from. Soliciting and processing bids is costly, and the per-bidder bid processing cost is κ . It includes the cost of reading the proposals, making sure that the language and terms of the proposals are unambiguous, and assessing various attributes of the contractors. The procurer can choose the rate at which bidders arrive and participate in the competition for the project, λ . Here, we assume that the risk of drawing no bids is zero, and that the realized number of participants follows a Poisson distribution shifted to the right by one.

¹In our paper, the extent to which the procurement agents, or the contracting officers, can manipulate the selection rule is very limited and the assignment of favor is exogenously determined. Alternatively, Burguet and Che (2004) consider a model whether a procurement agent can manipulate the quality assessment to favor the high briber, and analyze how corruption affects the allocation of contracts and the procurement costs. Celentani and Ganuza (2002), on the other hand,

Therefore, the total cost of the procurement, denoted as V , depends on the identity of the winning contractor and the exclusivity of the competitive procedure. The total procurement cost includes the bid processing cost and the private benefits from giving a contract to a favored agent, as well as the payment to the winning contractor. The procurer minimizes the expected value of this total cost when she chooses whether or not to focus her search to favored agents and the rate at which agents arrive for participation in the procurement.

The expected total cost of the procurement by holding an exclusive competition where the chosen bid arrival rate is λ is denoted as $V(\textit{exclusive}, \lambda)$.

$$V(\textit{exclusive}, \lambda) = \sum_{n=1}^{\infty} \mathbb{E}(T|n, 0) \frac{(\lambda - 1)^{n-1} e^{-\lambda+1}}{(n-1)!} + \kappa\lambda - \delta, \quad (8)$$

where $\mathbb{E}(T|n, 0)$ is the expected payment to the winning contractor when n favored bidders participate while no unfavored bidders do. The optimal rate of arrival of bidders, λ_e , minimizes the above expected total cost.

On the other hand, the expected total cost of the procurement when all bidders can participate and the chosen bid arrival rate is λ is as $V(\textit{nonexclusive}, \lambda)$.

$$V(\textit{nonexc.}, \lambda) = \sum_n \sum_{n_f+n_u=n} \mathbb{E}(T|n_f, n_u) \Pr(n_f, n_u; \lambda) + \kappa\lambda - \delta \Pr(\textit{favored}|\textit{nonexc.}, \lambda), \quad (9)$$

where $\mathbb{E}(T|n_f, n_u)$ is the the expected payment to the winning contractor when n_f favored bidders participate and n_u unfavored bidders do. $\Pr(n_f, n_u; \lambda)$ denotes the probability that n_f favored agents and n_u unfavored agents participate.

$$\Pr(n_f, n_u; \lambda) = \frac{(\lambda - 1)^{n-1} e^{-\lambda+1}}{(n-1)!} \binom{n}{n_f} \rho(n)^{n_f} (1 - \rho(n))^{n_u},$$

where $\rho(n)$ is the probability that a contractor is favored among n participants. The probability that the winner is a favored agent is denoted by $\Pr(\textit{favored}|\textit{nonexclusive}, \lambda)$, which can be written as:

$$\begin{aligned} \Pr(\textit{favored}|\textit{nonexc.}; \lambda) &= 1 - \sum_{n=1}^{\infty} \frac{(\lambda - 1)^{n-1} e^{-\lambda+1}}{(n-1)!} \sum_{n_f+n_u=n} [1 - \pi_f(n_f)]^{n_f} \\ &\quad [1 - (1 - \pi_u(n_u))^{n_u}] \binom{n}{n_f} \rho(n)^{n_f} (1 - \rho(n))^{n_u}. \quad (10) \end{aligned}$$

The optimal arrival rate for a nonexclusive competition, λ_o minimizes the above expected

total cost.

We assume that the procurer directly receives private benefit η by excluding unfavored agents from participation. Therefore, the procurer chooses to limit competition if the following condition holds:

$$V(\textit{exclusive}, \lambda_e) - \eta \leq V(\textit{nonexclusive}, \lambda_o).$$

4 Data and Descriptive Analysis

The above theory predicts the optimal extent of competition and contract terms. In the remainder of this paper, we analyze data from contracts let by the U.S. federal government. Our dataset provides a very detailed information on the extent of competition and the contract terms for each procurement contract. In this section, we provide a description on the source of the data and the variables for our analysis. Then we explore the determinants of the extent of competition and the total payment to the contractors.

4.1 Variables in the Data

The data is a selected set of contracts let by the U.S. federal government during the period of FY 2004–2012. It is sourced from Federal Procurement Data System - Next Generation, which collects information on contracts and their modifications. We restricted attention to contracts with specified terms and conditions of an expected size of \$300,000 or more, and then we further narrowed down our sample that satisfy the following criteria: (i) available for competition but commercially unavailable, (ii) competed under the competitive proposal evaluation procedure or other similar procedures, (iii) not set-aside for small business concerns, (iv) initiated and completed during the period of study, (v) performed in the 48 contiguous states in the U.S., (vi) expected to take longer than two weeks for completion, and (vi) without any inconsistent records on the contract. For our analysis, we focus on the procurement projects that are not commercially available. There are 33,392 contracts that satisfy all of the above criteria, totaling \$192 billion. For each contract, we construct the variables in Table 2.

The base payment of a contract is defined as the total value of the contract plus all options that have been exercised at the time of award. On the other hand, the final payment is the total amount of funds obligated to the government. The base payments and the final payments often differ from each other, and the final payments are on average larger than than the base payments. These differences are due to the contract terms that allow

Table 2: Summary Statistics: Federal Procurement Contracts, 2004–2012

	Mean	Std. Dev.	Min	Max
<i>Payments (\$K, 2010)</i>				
Base Payment	4530.7	83,909.4	1.0	13,892,030
Final Payment	5,766.7	47,294.3	-4,417.5	4,126,789
Payment Adjustment: Shock ^a	502.7	74,794.2	-13,392,210	1,182,744
Payment Adjustment: Signal ^b	2,229.9	30,049.5	-196,352	2,944,045
<i>Duration (Days)</i>				
Base Duration	432.7	362.5	15	3,643
Final Duration	688.4	580.5	0	3,929
Duration Change: Shock ^a	79.5	261.2	-2,397	3,186
Duration Change: Signal ^b	176.2	373.3	-2,284	3,223
<i>Contract Type</i>				
Fixed-Price Contract	0.5441	0.4980	0	1
<i>Competition</i>				
Exclusive Competition	0.5049	0.4999	0	1
Number of Offers	7.25 22	29.8375	1	730
<i>Contract Attributes</i>				
State Population Density (per mi^2)	1,009.02	2,514.69	1.15	10,357.46
State Per Capita Income (\$K, 2010)	41,116	92,992	21,811	78,262
# of Contractors in Industry/State	26.9653	41.8114	2	227
<i>Political Attributes</i>				
House Appropriations Committee Member	0.2046	0.4034	0	1
House Appropriations Committee Top 5	0.0190	0.1366	0	1
Abuse of Public Office Convictions ^c	0.11	0.20	0	1.39
Police Employment ^d	0.10	0.04	0.007	0.21
Monthly Work Overload	39.64	121.38	-99.69	843.53

* Note: The number of observations is 33,392. The amount of payments is CPI-adjusted, where CPI of December 2010 is 100. a. This adjustment is related to work requirement changes or other significant contract modifications. b. This adjustment includes all other adjustments. c. The total number of the conviction cases of public office abuse in the state of performance per one thousand government employees in the state during the year of signing the contract. d. The ratio of police employment to the total government employment in the state of performance.

the final payment to vary with the observed outcomes of the project. Some outcomes are correlated with the efficiency of the contractor, and others are not. We consider the work requirement changes are in the latter category, while the rest are in the former, which we consider as a signal in our model. We refer the payment changes due to a signal as “Ex-post Payment Adjustment” in Table 2. Note that the average amount of the payment adjustment is nonzero.

The contract types are grouped into two broad categories: fixed-price and variable-price. The specific contract types range from firm-fixed-price, in which the contractor has full responsibility for the performance costs and resulting profit or loss, to cost-plus-fixed-fee, in which the contractor has minimal responsibility for the performance costs and the negotiated fee is fixed. In between are the various incentive contracts, we consider firm-fixed-price contracts as fixed-price, and the rest as variable-price. In our sample, about 52% of the contracts are fixed-price.

The extent of competition, which can be determined by the contracting officers with discretion, is observed in two dimensions. One is whether or not there was a full and open competition, where no contractors were excluded from participation. 50% of the contracts in the sample were let under a full and open competition. The other is the number of actual participants. The average number of them are 7.33, but the median is 1.

The extent of *potential* or *underlying* competition is measured indirectly from two variables. One is the population density of the state where the project is to be performed when the contract was signed. The density is measured by the number of residents in the state of performance deflated by the squared miles of land area. The higher the population density is, the more likely that the market competition is fierce. The other variable is the number of the unique winning contractors of the industry during the year of signing of the contract.

Some contractors may have better political connections than others. A winning contractor may be represented by a member of the House Appropriations Committee. Lastly, we measure the extent of corruption at the location of the performance of the procurement by the number of public officials convicted for abuse of public office in the state deflated by total government employment. These data are drawn from the report to the *Congress on the Activities and Operations of the Public Integrity Section* published by the US Department of Justice. These data have been used to measure the extent of corruption at the state-level by Goel and Rich (1989) and Fisman and Gatti (2002), amongst others. Because higher conviction might indicate both more widespread corruption as well as a higher degree of enforcement of justice at the state level, we also collected the share of police employment over total government employment.

4.2 The Extent of Competition

We first study what determines the extent of competition for procurement. Table 3 shows two sets of regression analyses. In one set of Probit regressions, the dependent variable is whether or not the competition is exclusive. In the other set of Poisson regressions, the dependent variable is the number of bids minus one.² While whether or not the competition was exclusive is not significantly correlated with the winning bidder’s representation by the members of Congress in the House Appropriations Committees (HAC), the number of bidders are negatively correlated with it. Furthermore, if the contractor is represented by the top five members from the majority party in terms of the ranking in the House Appropriations Committee, this effect is much larger. We also find that the larger the number of convictions for public office abuse is, the more likely the contract is completed for exclusively.

4.3 The Payments to the Contractors

We study what determines the payment to the contractors. Table 4 shows two sets of regression analyses. We consider two types of payments. One is the total payment, and the other is the payment related to the cost shocks in the model, which occurs as a result of non-administrative contract modifications. While the winning bidder’s representation by the members of Congress in the House Appropriations Committees (HAC) and the measure of corruption are correlated with the extent of competition, they are not significantly correlated with either type of payments. This supports our modeling assumption that the procurer may favor certain contractors at the source selection stage, but the contracting terms are irrelevant to favoritism.

5 Identification

Suppose we observe a random sample of projects. For each i^{th} project, we observe the following variables on the extent of competition, the contract chosen by the winning contractor, and the realized payment. First, we observe the number of participants, N_i , and Y_i , an indicator variable which takes 1 if project i was subject to an exclusive competition, and 0 if it was nonexclusive. Second, we observe another indicator variable, W_i , which takes 1 if the winner chose a fixed-price contract and 0 otherwise. Third, we observe the base payment P_i , the final payment T_i , the ex-post payment adjustment due to a signal (Q_i), and the realized

²We subtract one from the number of bids when we fit a Poisson distribution to account for the fact that we do not have any observations with no bidders.

Table 3: Determinants of the Extent of Competition

Model	(1)	(2)	(3)	(4)
Dependent Variable	Probit Exclusive	Probit Exclusive	Possion # of Bidders	Possion # of Bidders
House Appro. Comm. Members	-0.0205 (0.0192)	-0.0223 (0.0193)	-0.0067 (0.0064)	-0.0131** (0.0064)
House Appro. Comm. Top 5 Members	-0.0519 (0.0545)	-0.0429 (0.0547)	-0.2939*** (0.0214)	-0.2695*** (0.0215)
Abuse of Public Office Convictions		0.2248** (0.0970)		0.4917*** (0.0321)
Police Employment		-4.6876*** (1.0461)		3.3962*** (0.3085)
Monthly Work Overload		0.0001* (0.00006)		-0.0018*** (0.00002)
ln(Base Duration)		0.0199** (0.0088)		0.1490*** (0.0033)
ln(Industry/State Avg. Base Payment)		-0.0548*** (0.0067)		-0.1103*** (0.0025)
# of Contractors in Industry/State		0.0016*** (0.0002)		0.0016*** (0.00005)
State Population Density		0.0005*** (0.0001)		-0.0006*** (0.00004)
State Per Capita Income		-0.0055 (0.0071)		0.0255*** (0.0020)
Industry, State, Year F.E.'s	Yes	Yes	Yes	Yes
Num. Obs.	33,392	33,118	33,392	33,118
R^2	0.0807	0.0856	0.2246	0.2363

* Note: The numbers in parentheses are standard errors. Asterisk marks represent the statistical significance: 10 (*), 5(**), and 1(***) percent.

Table 4: Determinants of the Payments to the Contractors

Model	(1)	(2)	(3)	(4)
Dependent Variable	OLS Total Payment	OLS Total Payment	OLS Adjustment Payment	OLS Adjustment Payment
House Appro. Comm. Members	-109.96 (703.15)	328.74 (689.26)	-253.15 (447.37)	-57.81 (445.12)
House Appro. Comm. Top 5 Members	-1,872.87 (2,004.30)	-1,434.81 (1,964.32)	-959.55 (1,275.22)	-753.76 (1,268.55)
Abuse of Public Office Convictions		-3,649.91 (3,491.71)		-2,349.95 (2,254.95)
Police Employment		-48,891.03 (35,754.62)		-36,584.68 (23,090.35)
Monthly Work Overload		-2.2866 (2.2136)		-3.1941** (1.4295)
ln(Base Duration)		4,911.54*** (317.05)		2,285.07*** (206.64)
ln(Industry/State Avg. Base Payment)		9,125.73*** (239.05)		3,910.65*** (154.38)
# of Contractors in Industry/State		-40.2373*** (7.6189)		-16.6468*** (4.9203)
State Population Density		-0.5960 (5.1287)		0.0829 (3.3121)
State Per Capita Income		621.98*** (249.99)		442.28*** (161.44)
Industry, State, Year F.E.'s	Yes	Yes	Yes	Yes
Num. Obs.	33,392	33,118	33,392	33,118
R^2	0.0038	0.0588	0.0031	0.0281

* Note: The numbers in parentheses are standard errors. Asterisk marks represent the statistical significance: 10 (*), 5(**), and 1(***) percent. The total payment to the contractor is the dependent variable for the first two regressions, and the payment related to the non-administrative contract modifications is the dependent variable for the last two regressions.

signal s_i . Notice that we do not observe the bids of participants other than the winner and the identity of the bidders that are favored by the procurer.

Given the variables that we observe in the data, we identify all components in the model. The distribution of the ex-post signal conditional on the efficiency of the contractor, $f(\cdot|\beta)$, can be directly identified from the observed distribution of the signal conditional on the contract type. As shown in Theorem 1, the efficient type chooses a fixed-price contract, and the inefficient type chooses a variable-price contract. Therefore, we identify $f(\cdot|\underline{\beta})$ from the observed distribution of the signal for fixed-price contracts and $f(\cdot|\bar{\beta})$ from that for variable-price contracts.

The ratio of efficient agents among n favored participants, denoted by $\pi_f(n)$, are identified from the probability that a procurement contract, competed by n bidders when the competition was exclusive to favored bidders only, is fixed-price, denoted by $\Pr(\text{fixed}|n, 0)$ for any $n \geq 1$. As can be seen in equation (4), there exists a one-to-one mapping between $\Pr(\text{fixed}|n, 0)$ and $\pi_f(n)$. This enables us to identify $\pi_f(n)$ from the observed $\Pr(\text{fixed}|n, 0)$.

The characterization of the variable-price contracts when the competition is exclusive helps us identify the liquidity cost function, $\psi(\cdot)$. Equation (7) can be written as:

$$\frac{1}{\psi'(\bar{q}(s; n))} = 1 + \frac{1 - n(1 - \pi_f(n))\bar{\phi}(n)}{n(1 - \pi_f(n))\underline{\phi}(n)} \left[1 - \frac{f(s|\underline{\beta})}{f(s|\bar{\beta})} \right],$$

for any $n \geq 1$ participants. The right hand side of the above equation is a function of $\pi_f(n)$ and $f(s|\beta)$, which has been identified earlier. The ex-post payment adjustment schedule in the exclusive contracts with n favored bidders, $\bar{q}(s; n)$, is observed in the data. Therefore, by exploiting the above equation, we identify $\psi'(\cdot)$, and the assumption that $\psi(0) = 0$ pins down $\psi(\cdot)$.

We briefly describe the identification argument for the rest of the components of the model, and the details are followed below. First, the identification of the ratio of efficient agents among n unfavored participants, denoted by $\pi_u(n)$, and the ratio of favored agents among n participants in nonexclusive competitions, denoted by $\rho(n)$, is complicated because we do not observe whether or not a bidder in a nonexclusive competition is favored. We identify them by exploiting their equilibrium relationship with the probability that a procurement contract is fixed-price and the characterization of a variable-price contract when the contract was competed without any restrictions in entry. Second, the distribution of the cost of completing a project for an efficient contractor, denoted by $G(\cdot)$, and the cost difference between an efficient contractor and an inefficient one, $\bar{\beta}$, are identified from the distribution of the base payments of contracts won in exclusive competitions. Lastly, the per-bidder processing cost, κ , is identified from the observed bidder arrival rate in exclusive

competitions, and the benefit from choosing a favored agent as a winner of a project, δ , is identified from the ratio of exclusive competition.

The Ratio of Favored Contractors in Nonexclusive Competition To identify the probability that a bidder is favored by the procurer when the bidder is the only participant, $\rho(1)$, we focus on the variable-price contracts won in a nonexclusive competition when a single bidder participated. Such contract is designed for either a favored contractor or an unfavored contractor. For any s , the observed ex-post payment adjustment in response to s is a weighted average of the two contracts: one for a favored contractor, denoted as $\bar{q}_f(s)$, and the other for an unfavored contractor $\bar{q}_u(s)$:

$$\bar{q}(s) = \frac{\rho(1)(1 - \pi_f(1))}{\Pr(\text{variable}|\text{nonexc.}, 1)}\bar{q}_f(s) + \frac{(1 - \rho(1))(1 - \pi_u(1))}{\Pr(\text{variable}|\text{nonexc.}, 1)}\bar{q}_u(s). \quad (11)$$

We recover $\bar{q}_f(\cdot)$ from the observed ex-post payment adjustment for the variable-price contracts won in an exclusive competition with a single bidder. However, $\bar{q}_u(\cdot)$ is not directly observed because we do not know which contractor is favored by the procurer. Instead, we solve for $\bar{q}_u(\cdot)$ using Theorem 2 as follows:

$$\bar{q}_u(s) = \begin{cases} \psi'^{-1} \left(\frac{1 - \pi_u(1)}{1 - \pi_u(1)} \frac{f(s|\beta)}{f(s|\beta)} \right) & \text{if } \frac{f(s|\beta)}{f(s|\beta)} < \frac{1}{\pi_u(1)}, \\ M & \text{otherwise.} \end{cases} \quad (12)$$

In the right hand side of the above equation, $f(\cdot|\beta)$ and $\psi'(\cdot)$ have been identified. The probability that one single unfavored participant is efficient, $\pi_u(1)$, can be solved as a function of $\rho(1)$ from the following equation on the probability that a winning contract is fixed-price when the competition is nonexclusive and only one bidder participates:

$$\begin{aligned} \Pr(\text{fixed}|\text{nonexc.}, 1) &= 1 - \Pr(\text{variable}|\text{nonexc.}, 1) \\ &= \rho(1)\pi_f(1) + (1 - \rho(1))\pi_u(1). \end{aligned}$$

Because $\Pr(\text{fixed}|\text{nonexclusive}, 1)$ is observed in the data and $\pi_f(1)$ have been identified, $\pi_u(1)$ can be written as a function of $\rho(1)$.

$$\pi_u(1) = \frac{\Pr(\text{fixed}|\text{nonexc.}, 1) - \rho(1)\pi_f(1)}{1 - \rho(1)}. \quad (13)$$

Substituting the above expression for $\pi_u(1)$ in equation (12), $\bar{q}_u(s)$ becomes a function of $\rho(1)$, which we denote by $\bar{q}_u(s; \rho(1))$. In the following proposition, we show that there

exists a unique $\rho(1)$ that satisfies equations (11) and (13) for s with positive probability if $\pi_f(1) \neq \pi_u(1)$ and the support of s is convex.

Proposition 1 *If $\pi_f(1) \neq \pi_u(1)$ and the support of s is convex, $\rho(1)$ is uniquely identified.*

Similarly, we identify $\rho(n)$ for $n > 1$.

The Ratio of Efficient Agents among Unfavored Participants Given that we identify $\rho(1)$, the probability that a single unfavored bidder is efficient, $\pi_u(1)$, is identified from equation (13). For $n > 1$, the probability can be identified from the observed probability that a winning contract is fixed-price under a nonexclusive competition with n bidders:

$$\Pr(\text{fixed}|\text{nonexc.}, n) = \sum_{n_f=0}^n \binom{n}{n_f} \rho(n)^{n_f} (1 - \rho(n))^{n_u} [1 - (1 - \pi_f(n_f))^{n_f} (1 - \pi_u(n_u))^{n_u}],$$

where $n_u \equiv n - n_f$. Suppose we have identified $\{\pi_u(1), \dots, \pi_u(n-1)\}$. Note that in the above equation, the left hand side is directly observed in the data, and we have identified $\rho(n)$, $\pi_f(n_f)$ for all n_f , and $\pi_u(n_u)$ for all $n_u < n$. This yields that the only unknown variable is $\pi_u(n)$ in the above equation, and the right hand side of the equation is increasing in $\pi_u(n)$. Therefore, $\pi_u(n)$ is identified. This completes the proof that $\pi_u(n)$ is identified for all n .

The Distribution of Project Costs to Contractors The observed distribution of the base payments of contracts won in exclusive contracts helps us identify the distribution of $c + \bar{\beta}$. In equation (5) in Theorem 4, the optimal base payment for a fixed-price contract is a function of c and $\bar{\beta}$, and using the variation in the number of bidders, we can separately identify the distribution of c and $\bar{\beta}$.

The Per-Bidder Bid Processing Cost We identify the bid process cost per bidder, κ , from the observed arrival rate of bidders under exclusive competition and the related optimality condition, denoted by λ_e . The rate of bidder arrivals minimizes the expected total cost of procurement to the procurer when the competition is limited, which is represented in equation (8). At the optimal arrival rate, the derivative of the expected total cost of procurement with respect to the arrival rate must be zero. This first order condition can be written as:

$$\sum_{n=1}^{\infty} \frac{\lambda_e^{n-2} e^{-\lambda_e+1}}{(n-1)!} (n - \lambda_e) \mathbb{E}(T|n, 0; c) + \kappa(c) = 0,$$

where $\mathbb{E}(T|n, 0; c)$ denotes the expected payment to a winning contractor when n favored bidders participate in the exclusive competition for a project with minimum cost c . Note

that because $\partial \mathbb{E}(T|n, 0; c) / \partial c = 1$ for all n , λ_e does not vary with c . Given that $\mathbb{E}(T|n, 0; c)$ for any c and λ_e are directly observed from the data, we identify κ using the above equation.

The Benefit from Selecting a Favored Contractor The benefit of selecting a favored bidder as a function of the minimum cost of a project, δ , can be identified from the probability that a project is subject to an exclusive competition as opposed to a nonexclusive competition. The procurer chooses to hold an exclusive competition for a project with the minimum cost c if and only if

$$V(\text{nonexc.}, \lambda_o; c) - V(\text{exclusive}, \lambda_e; c) + \eta \geq 0,$$

where $V(\text{exclusive}, \lambda_e; c)$ and $V(\text{nonexclusive}, \lambda_o; c)$ denote the expected total cost of procurement with or without restrictions in entry respectively where the optimal bidder arrival rates, λ_o and λ_e , are chosen. We assume that the private benefit of holding an exclusive competition as opposed to a nonexclusive competition, η , is drawn from the standard logistic distribution. Given this assumption, we have the following equation:

$$\log \frac{\Pr(\text{nonexc.}|c)}{\Pr(\text{exclusive}|c)} = V(\text{exclusive}, \lambda_e; c) - V(\text{nonexc.}, \lambda_o; c).$$

Note that $V(\text{exclusive}, \lambda_e; c) - V(\text{nonexc.}, \lambda_o; c)$ does not depend on c , implying that the left hand side is also independent of c . Using equations (8) and (9), the right hand side of the above equation can be expanded as follows:

$$\begin{aligned} \log \frac{\Pr(\text{nonexc.})}{\Pr(\text{exclusive})} &= \mathbb{E}[T(\text{exclusive}) - T(\text{nonexc.})] + \kappa[\lambda_e - \lambda_o] \\ &- \delta[1 - \Pr(\text{favored}|\text{nonexc.}, \lambda_o)], \end{aligned}$$

where $\mathbb{E}[T(\text{nonexclusive}) - T(\text{exclusive})]$ denotes the expected difference in the payment to a winning contractor. $\Pr(\text{favored}|\text{nonexclusive}, \lambda_o)$ denotes the probability that the winner of a procurement project in a nonexclusive competition is a favored agent when the average bidder arrival rate is λ_o , which is defined in equation (10). Note that all arguments except δ in the above equation are either directly observed from the data ($\Pr(\text{nonexclusive})$, $\Pr(\text{exclusive})$, $\mathbb{E}[T(\text{nonexclusive}) - T(\text{exclusive})]$, and $\lambda_o - \lambda_e$) or have been identified (κ and $\Pr(\text{favored}|\text{nonexclusive}, \lambda_o)$). Therefore, we uniquely identify δ .

6 Results

6.1 Nonparametric Estimator

We closely follow the identification arguments to construct the nonparametric estimator. We assume that all components of the model to be estimated depend on the observed characteristics of a procurement project. These characteristics, denoted by \mathbf{X}_i , include (i) the number of unique bidders that won any of the procurement contracts of the industry during the period of study, (ii) the average base payments of the contracts of the industry during the same period, (iii) the base duration of the project, (iv) the ratio of conviction of public office abuse to the number of the government employees of the year when the contract was signed in the state where the contract is performed, and (v) the ratio of police force to the number of the government employees of the year and the state.

The Distribution of the Ex-post Signal Because if two contracts are offered to agents, an efficient agent ($\beta = \underline{\beta}(\mathbf{x})$) chooses a fixed-price contract and an inefficient agent ($\beta = \overline{\beta}(\mathbf{x})$) chooses a variable-price contract in equilibrium. Therefore, we can estimate the conditional distributions of the ex-post signal from the observed distribution of the realized signal for fixed-price contracts and that for variable-price contracts respectively. For example, we use a kernel density estimator for the signal distribution of an inefficient type as follows:

$$\hat{f}(s|\underline{\beta}, \mathbf{x}) = \frac{\sum_i w_i K_s\left(\frac{s_i - s}{h_s}\right) K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}{h_1 \sum_i w_i K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)},$$

where $K_s(\cdot)$ and $K_x(\cdot)$ are the Gaussian kernel functions and $h_s > 0$ and $h_x > 0$ are bandwidths based on the “normal reference rule-of-thumb” approach as suggested by Silverman (1986).

The Ratio of Efficient Contractors among Favored Participants To estimate $\pi_f(n, \mathbf{x})$, we first estimate the probability that a fixed-price contract is administered under an exclusive competition with n bidders for $n = 1, 2, \dots$:

$$\Pr(\widehat{fixed|exclusive}, n, \mathbf{x}) = \frac{\sum_i w_i y_i 1\{n_i = n\} K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}{\sum_i y_i 1\{n_i = n\} K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}.$$

Given $\Pr(\widehat{fixed|exclusive}, n, \mathbf{x})$, we estimate $\hat{\pi}_f(n, \mathbf{x})$ for any $n \geq 1$ as follows:

$$\hat{\pi}_f(n, \mathbf{x}) = 1 - \left(1 - \Pr(\widehat{fixed|exclusive}, n, \mathbf{x})\right)^{1/n}.$$

The Liquidity Cost Function and the Penalty Function In estimating the liquidity cost function, we focus on the variable-price contracts with entry restrictions. When n bidders participate, the ex-post payment adjustment $\bar{q}_f(\cdot; \mathbf{x}, n)$ satisfies the characterization in Theorem 4. We allow that observed payment adjustment may include some part of the actual base payment, denoted by $\gamma(\mathbf{x})$. Here, we jointly estimate $\psi(\cdot; \mathbf{x})$ and $\gamma(\mathbf{x})$ by the following procedure.

Using equation (7) in the theorem, we first derive $\hat{\psi}'(q_i + p_0(\mathbf{x}); \mathbf{x}) \equiv \psi_i$ for each observed contract i which is variable-price competed under entry restrictions. For any signal s_i such that $\frac{f(s_i|\underline{\beta}(\mathbf{x}))}{f(s_i|\bar{\beta}(\mathbf{x}))} < \frac{1}{\pi_f(n, \mathbf{x})}$:

$$\psi_i = \left(1 + \frac{1 - n_i(1 - \hat{\pi}_f(n_i; \mathbf{x}))\hat{\phi}_f(n_i; \mathbf{x})}{n_i(1 - \hat{\pi}_f(n_i; \mathbf{x}))\underline{\phi}_f(n_i; \mathbf{x})} \left[1 - \frac{\hat{f}(s_i|\underline{\beta}(\mathbf{x}))}{\hat{f}(s_i|\bar{\beta}(\mathbf{x}))}\right]\right)^{-1}.$$

We then estimate $\tilde{\psi}'(q; \mathbf{x}) \equiv \hat{\psi}'(q + \gamma(\mathbf{x}); \mathbf{x})$ using a Nadaraya-Watson estimator:

$$\tilde{\psi}'(q; \mathbf{x}) = \frac{\sum_i \psi_i y_i (1 - w_i) K_q\left(\frac{q_i - q}{h_q}\right) K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}{\sum_i y_i (1 - w_i) K_q\left(\frac{q_i - q}{h_q}\right) K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)},$$

where $K_q(\cdot)$ is the Gaussian kernel function and h_q is a bandwidth. Exploiting our assumption that $\psi'(0; \mathbf{x}) = 1$,

$$\hat{\gamma}(\mathbf{x}) = (\tilde{\psi}')^{-1}(1; \mathbf{x}).$$

Lastly, the assumption $\psi(0; \mathbf{x}) = 0$, we have

$$\hat{\psi}(q; \mathbf{x}) = \int_0^q \tilde{\psi}'(u - \hat{\gamma}(\mathbf{x}); \mathbf{x}) du.$$

As for the penalty function, we estimate the sample average of $q_i - \hat{\gamma}(\mathbf{x})$ for s_i such that $\frac{f(s_i|\underline{\beta}(\mathbf{x}))}{f(s_i|\bar{\beta}(\mathbf{x}))} \geq \frac{1}{\pi_f(n, \mathbf{x})}$.

$$\hat{M}(\mathbf{x}) = \frac{\sum_i (q_i - \hat{\gamma}(\mathbf{x})) y_i (1 - w_i) \mathbf{1}\left\{\frac{\hat{f}(s_i|\underline{\beta}(\mathbf{x}))}{\hat{f}(s_i|\bar{\beta}(\mathbf{x}))} \geq \frac{1}{\pi_f(n, \mathbf{x})}\right\} K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}{\sum_i y_i (1 - w_i) \mathbf{1}\left\{\frac{\hat{f}(s_i|\underline{\beta}(\mathbf{x}))}{\hat{f}(s_i|\bar{\beta}(\mathbf{x}))} \geq \frac{1}{\pi_f(n, \mathbf{x})}\right\} K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}.$$

The Ratio of Favored Contractors in Nonexclusive Competition and the Ratio of Efficient Contractors among Unfavored Participants

When the competition is non-exclusive, the ex-post payment adjustment schedule given n participants, $\bar{q}(s; n, \mathbf{x})$, could vary by the number of favored participants, n_f . Let us denote each schedule by $\tilde{q}(\cdot; n_f, n, \mathbf{x})$, and the probability that there are n_f favored contractors among n participants given that the contract is variable-price by $\Pr(n_f|n, \text{variable}, \mathbf{x})$. Given these notations, the following equation must hold in equilibrium for all s :

$$\bar{q}(s; n, \mathbf{x}) - \sum_{n_f=0}^n \Pr(n_f|n, \text{variable}, \mathbf{x}) \tilde{q}(s; n_f, n, \mathbf{x}) = 0.$$

Based on the above equation, we propose an estimator of $\rho(n, \mathbf{x})$ for each n :

$$\hat{\rho}(n, \mathbf{x}) = \arg \min_{\rho \in [0,1]} \sum_{j=1}^{n_s} \left[\hat{q}(s_j; n, \mathbf{x}) - \sum_{n_f=0}^n \Pr(n_f|\widehat{n, \text{var.}}, \mathbf{x}, \rho) \hat{q}(s_j; n_f, n, \mathbf{x}, \rho) \right]^2 \hat{f}(s_j|\bar{\beta}(\mathbf{x})),$$

where the outer summation is over a grid of signals, s_j , of size n_s . Below we define $\hat{q}(s_j; n, \mathbf{x})$, $\Pr(n_f|\widehat{n, \text{variable}}, \mathbf{x}, \rho)$, and $\hat{q}(s_j; n_f, n, \mathbf{x}, \rho)$. First, for a given signal s_j , we estimate $\hat{q}(s_j; n, \mathbf{x})$ using a Nadaraya-Watson estimator.

$$\hat{q}(s_j; n, \mathbf{x}) = \frac{\sum_i (q_i - \hat{\gamma}(\mathbf{x})) (1 - y_i) (1 - w_i) K_s \left(\frac{s_i - s_j}{h_s} \right) K_x \left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x} \right)}{\sum_i (1 - y_i) (1 - w_i) K_s \left(\frac{s_i - s_j}{h_s} \right) K_x \left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x} \right)}.$$

Note that we adjust the observed q_i by $\hat{\gamma}(\mathbf{x})$ in the above equation.

Second, the probability that there are n_f favored contractors among n participants given that the contract is variable-price and competed non-exclusively, $\Pr(n_f|n, \text{variable}, \mathbf{x}, \rho)$, is estimated as follows:

$$\Pr(n_f|\widehat{n, \text{var.}}, \mathbf{x}, \rho) = \frac{\binom{n}{n_f} \rho^{n_f} (1 - \rho)^{n - n_f} (1 - \hat{\pi}_f(n_f, \mathbf{x}))^{n_f} (1 - \hat{\pi}_u(n - n_f, \mathbf{x}, \rho))^{n - n_f}}{\sum_{n_f=0}^n \binom{n}{n_f} \rho^{n_f} (1 - \rho)^{n - n_f} (1 - \hat{\pi}_f(n_f, \mathbf{x}))^{n_f} (1 - \hat{\pi}_u(n - n_f, \mathbf{x}, \rho))^{n - n_f}}.$$

In the above equation, $\hat{\pi}_f(n_f, \mathbf{x})$ has been identified for any n_f . The probability that an unfavored contractor is efficient among n of them, $\hat{\pi}_u(n, \mathbf{x}, \rho)$, is function of ρ , as defined as:

$$\begin{aligned} & \hat{\pi}_u(n, \mathbf{x}, \rho) \\ = & 1 - \left[1 - \frac{\left\{ \widehat{\Pi}_n(\mathbf{x}) - \sum_{k=1}^n \binom{n}{k} \rho^k (1 - \rho)^{n-k} [1 - (1 - \hat{\pi}_f(k, \mathbf{x}))^k (1 - \hat{\pi}_u(n - k, \mathbf{x}))^{n-k}] \right\}}{(1 - \rho)^n} \right]^{\frac{1}{n}}, \end{aligned}$$

where

$$\widehat{\Pi}_n(\mathbf{x}) \equiv \Pr(\widehat{fixed}|\widehat{nonexc.}, n_u, \mathbf{x}) = \frac{\sum_i w_i(1 - y_i)1\{n_i = n_u\}K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}{\sum_i (1 - y_i)1\{n_i = n_u\}K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}.$$

Note that in the above equation for $\hat{\pi}_u(n_u, \rho)$, $\hat{\pi}_u(k, \mathbf{x})$ for $k < n_u$ are the estimates of $\pi_u(k, \mathbf{x})$ based on the estimates of $\rho(k, \mathbf{x})$ and $\pi_u(k, \mathbf{x})$:

$$\hat{\pi}_u(n, \mathbf{x}) \equiv \hat{\pi}_u(n, \mathbf{x}, \hat{\rho}(n, \mathbf{x})).$$

Therefore, the estimation procedure must be sequential in the order of the number of total participants. We start from estimating $\rho(1, \mathbf{x})$ and $\pi_u(1, \mathbf{x})$, based on which we estimate $\rho(2, \mathbf{x})$ and $\pi_u(2, \mathbf{x})$.

Lastly, we define $\hat{q}(s_j; n_f, n, \mathbf{x}, \rho)$ based on the characterization of the ex-post payment adjustment schedule in equation (17) in Theorem 5. If s_j is such that the first order condition holds,

$$\hat{q}(s_j; n_f, n, \mathbf{x}, \rho) = (\hat{\psi}')^{-1} \left(\left(1 + \frac{\hat{\varphi}_1(n_f, \mathbf{x})}{\hat{\varphi}_2(n_f, n - n_f, \mathbf{x}, \rho)} \frac{\hat{\phi}(n_f, n - n_f, \mathbf{x}, \rho)}{\hat{\phi}(n_f, \mathbf{x})} \left[1 - \frac{\hat{f}(s_j|\underline{\beta})}{\hat{f}(s_j|\bar{\beta})} \right] \right)^{-1} \right).$$

Otherwise, $\hat{q}(s_j; n_f, n, \mathbf{x}, \rho) = \hat{M}(\mathbf{x})$. In the following, we define $\hat{\varphi}_1(n_f, \mathbf{x})$, $\hat{\varphi}_2(n_f, n_u, \mathbf{x}, \rho)$, $\hat{\phi}(n_f, \mathbf{x})$, and $\hat{\phi}(n_f, n_u, \mathbf{x}, \rho)$ in the above equation for $1 \leq n_f \leq n - 1$:

$$\hat{\varphi}_1(n_f, \mathbf{x}) \equiv \Pr(\widehat{fixed}|\widehat{favored}, n_f) = 1 - (1 - \hat{\pi}_f(n_f, \mathbf{x}))^{n_f},$$

$$\hat{\varphi}_2(n_f, n_u, \mathbf{x}, \rho) \equiv \Pr(\widehat{var.}|\widehat{favored}, n_f, n_u) = (1 - \hat{\pi}_f(n_f, \mathbf{x}))^{n_f} (1 - \hat{\pi}_u(n_u, \mathbf{x}, \rho))^{n_u},$$

$$\hat{\phi}(n_f, \mathbf{x}) = \sum_{k=0}^{n_f-1} \binom{n_f-1}{k} \frac{\hat{\pi}_f(n_f, \mathbf{x})^k (1 - \hat{\pi}_f(n_f, \mathbf{x}))^{n-1-k}}{k+1},$$

$$\hat{\phi}(n_f, n_u, \mathbf{x}, \rho) = \frac{(1 - \hat{\pi}_f(n_f, \mathbf{x}))^{n_f-1} (1 - \hat{\pi}_u(n_u, \mathbf{x}, \rho))^{n_u}}{n_f}.$$

When $n_f = 0$ or $n_f = n$, i.e., all n participants are homogeneous in terms of favor, we define $\hat{q}(s_j; n_f, n, \mathbf{x}, \rho)$ based on equation (16) in Theorem 4.

The Distribution of Project Costs to Contractors We assume that at least one of the control variables is independent of the distribution of c .

Assumption 1 $G(\cdot|\mathbf{X}) = G(\cdot|\tilde{\mathbf{X}})$, where $\tilde{\mathbf{X}}$ denotes a subvector of \mathbf{X} .

Under this assumption, we use observations with $\tilde{\mathbf{X}}_i = \tilde{\mathbf{x}}$ to estimate the distribution of project costs.

A sample analogue of the characterization of the base payment of the fixed-price contracts when the competition is exclusive as in Theorem 4 can be written as:

$$p_i = c_i + \frac{\hat{\phi}(n_i, \mathbf{x}_i)}{\underline{\hat{\phi}}(n_i, \mathbf{x}_i)} \left[\bar{\beta}(\tilde{\mathbf{x}}) - \sum_j \hat{\psi}(\hat{q}(s_j; n_i, \mathbf{x}_i)) \left[\hat{f}(s_j | \bar{\beta}(\tilde{\mathbf{x}})) - \hat{f}(s_j | \underline{\beta}(\tilde{\mathbf{x}})) \right] |s_j - s_{j-1}| \right].$$

for any fixed-price, exclusive contract with $\tilde{\mathbf{X}} = \tilde{\mathbf{x}}$. Given the above equation, we separately estimate $\bar{\beta}(\tilde{\mathbf{x}})$ and $G(\cdot; \tilde{\mathbf{x}})$.

The Per-Bidder Bid Processing Cost To estimate the bid process cost per bidder, $\kappa(\mathbf{x})$, we exploit that the bidder arrival rate when the competition is excluded, $\lambda_e(\mathbf{x})$ minimizes the expected total cost of procurement to the procurer when the competition is limited, $V(\text{exclusive}, \lambda_e(\mathbf{x}), c; \mathbf{x})$. We first estimate $\lambda(\mathbf{x})$ by:

$$\hat{\lambda}_e(\mathbf{x}) = \frac{\sum_i n_i y_i K_x \left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x} \right)}{\sum_i y_i K_x \left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x} \right)}.$$

Because $\lambda(\mathbf{x})$ must minimize $V(\text{exclusive}, \lambda_e(\mathbf{x}), c; \mathbf{x})$, we estimate $\hat{\kappa}(\mathbf{x})$ using a sample analogue of the corresponding first order condition.

$$\hat{\kappa}(\mathbf{x}) = - \sum_{n=1}^{\infty} \frac{(\hat{\lambda}_e(\mathbf{x}) - 1)^{n-2} e^{-\hat{\lambda}_e(\mathbf{x})+1}}{(n-1)!} (n - \hat{\lambda}_e(\mathbf{x})) \mathbb{E}(\widehat{T|n, 0; \mathbf{x}}),$$

where the expected payment to a winner of an exclusive competition with n bidders, $\mathbb{E}(T|n, 0; \mathbf{x})$, is estimated as follows:

$$\mathbb{E}(\widehat{T|n, 0; \mathbf{x}}) = \frac{\sum_i t_i y_i 1\{n_i = n\} K_x \left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x} \right)}{\sum_i y_i 1\{n_i = n\} K_x \left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x} \right)}.$$

The Benefit from Selecting a Favored Contractor To estimate the benefit of selecting a favored bidder, $\delta(\mathbf{x})$, we exploit the model prediction on the probability that a project is subject to an exclusive competition as opposed to a nonexclusive competition. It can be shown that

$$\delta(\mathbf{x}) = \frac{\log \frac{\Pr(\text{nonexc.}; \mathbf{x})}{\Pr(\text{exclusive}; \mathbf{x})} + \mathbb{E}[T(\text{nonexc.}; \mathbf{x}) - T(\text{exclusive}; \mathbf{x})] + \kappa(\mathbf{x})[\lambda_o(\mathbf{x}) - \lambda_e(\mathbf{x})]}{1 - \Pr(\text{favored}|\text{nonexc.}; \mathbf{x})}.$$

By replacing all objects in the right hand side of the above equation with their sample analogues, we obtain an estimator of $\delta(\mathbf{x})$ as follows. The sample analogues of the most items are straightforward, except $\Pr(\widehat{favored|nonexc.}; \mathbf{x})$

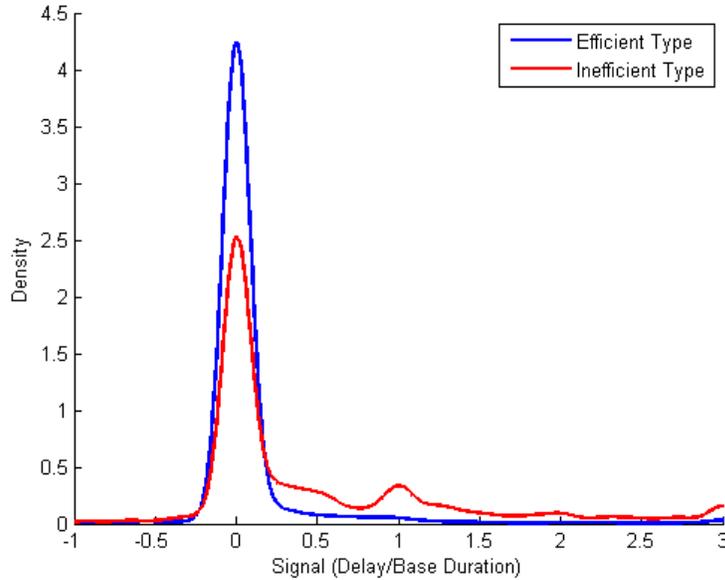
$$\Pr(\widehat{favored|nonexc.}; \mathbf{x}) = 1 - \sum_{n=1}^{\infty} \frac{(\hat{\lambda}_o(\mathbf{x}) - 1)^{n-1} e^{-\lambda_o(\mathbf{x})+1}}{(n-1)!} \sum_{n_f=0}^n [1 - \hat{\pi}_f(n_f, \mathbf{x})]^{n_f} [1 - (1 - \hat{\pi}_u(n - n_f, \mathbf{x}))^{n-n_f}] \binom{n}{n_f} \hat{\rho}(\mathbf{x})^{n_f} (1 - \hat{\rho}(\mathbf{x}))^{n-n_f}.$$

6.2 Estimates of the Components of the Model

We provide the estimation results conditional on the median value of the observed characteristics of a procurement project. The median values of these characteristics are (i) 11 unique bidders that won any of the procurement contracts of the industry during the period of study, (ii) \$1.1 million of the average base payments of the contracts of the industry during the same period, (iii) 364 days of base duration, (iv) 0.58 cases of conviction of public office abuse per 1,000 policemen of the year when the contract was signed in the state where the contract is performed. The standard errors will be provided in the later version of this draft.

First, the distribution of the delay divided by the base duration varies by type, as shown in Figure 2.

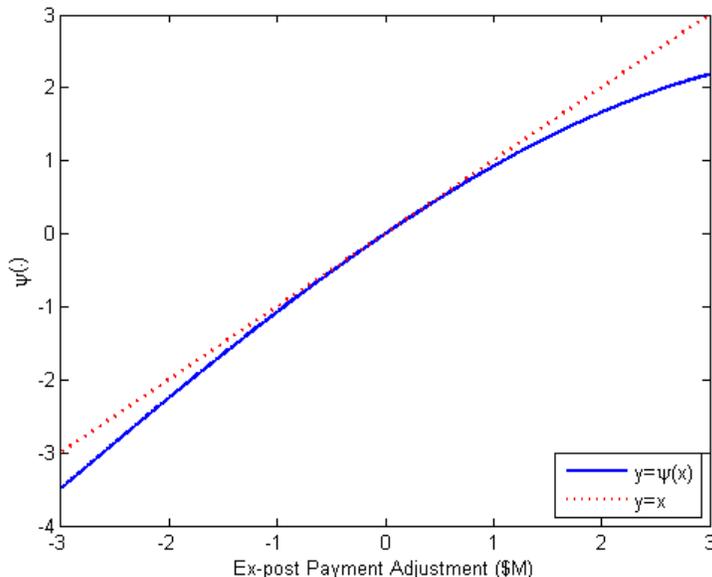
Figure 2: Signal Distributions by Type



Second, the estimated liquidity cost function, $\psi(\cdot)$, and its derivative can be found in

Figure 3. The liquidity cost function is estimated to be increasing and concave.

Figure 3: Liquidity Cost Function, $\psi(\cdot)$



Third, the average efficiency of bidders decreases as the number of bidders increases. This pattern is observed for both favored and unfavored contractors, as shown in figure 4.

Fourth, as the number of participants increases in a nonexclusive competition, the ratio of favored contractors among the participants drops. Figure 5 represents this pattern.

Lastly, the private benefit of hiring a favored contractor is estimated to be \$430,590, which is 11.65% of the average payment to the winning contractors, \$3.694 million. The bid processing cost is \$79,753 per bidder, or 2% of the average payment. Table 5 shows how the estimates of these parameters change as the ratio of conviction cases of public office abuse to the police size changes.

Table 5: Estimates of $\delta(\mathbf{x})$ and $\kappa(\mathbf{x})$

Conditional on Corruption Level	Median	25 Percentile	75 Percentile
Private Benefit from Favoritism (δ)	\$430,590	\$639,480	\$773,430
Per-bidder Bid Processing Cost (κ)	\$79,753	\$74,113	\$80,122

* Note: The estimates are conditional on the median values of the other observed characteristics. The median number of conviction cases of public office abuse per 1,000 policemen is 0.58, 25% being 0.33, and the standard deviation is 1.32.

Figure 4: Ratio of Efficient Contractors by # of Bidders

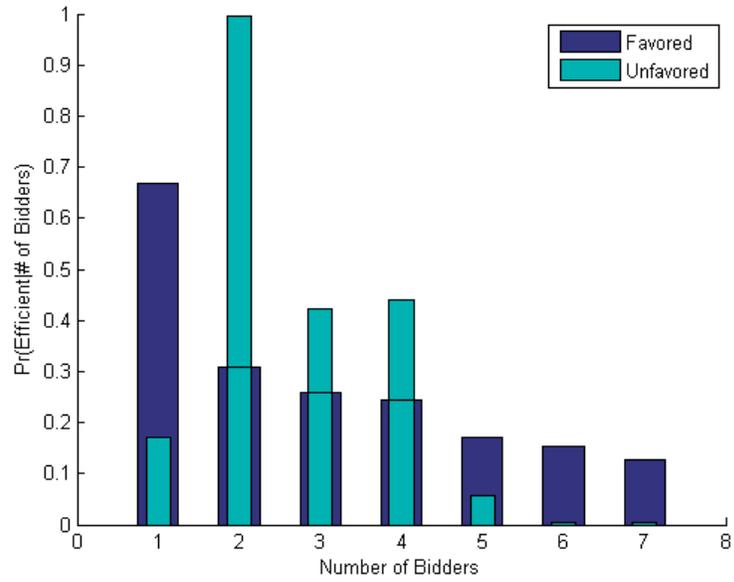
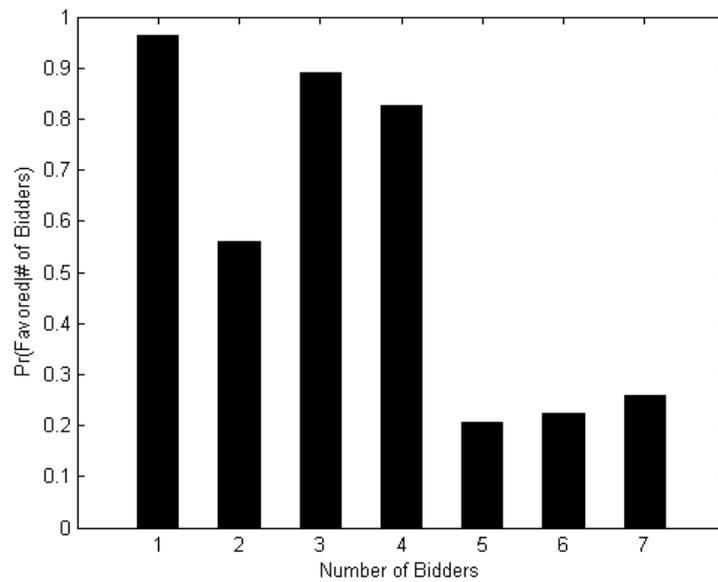


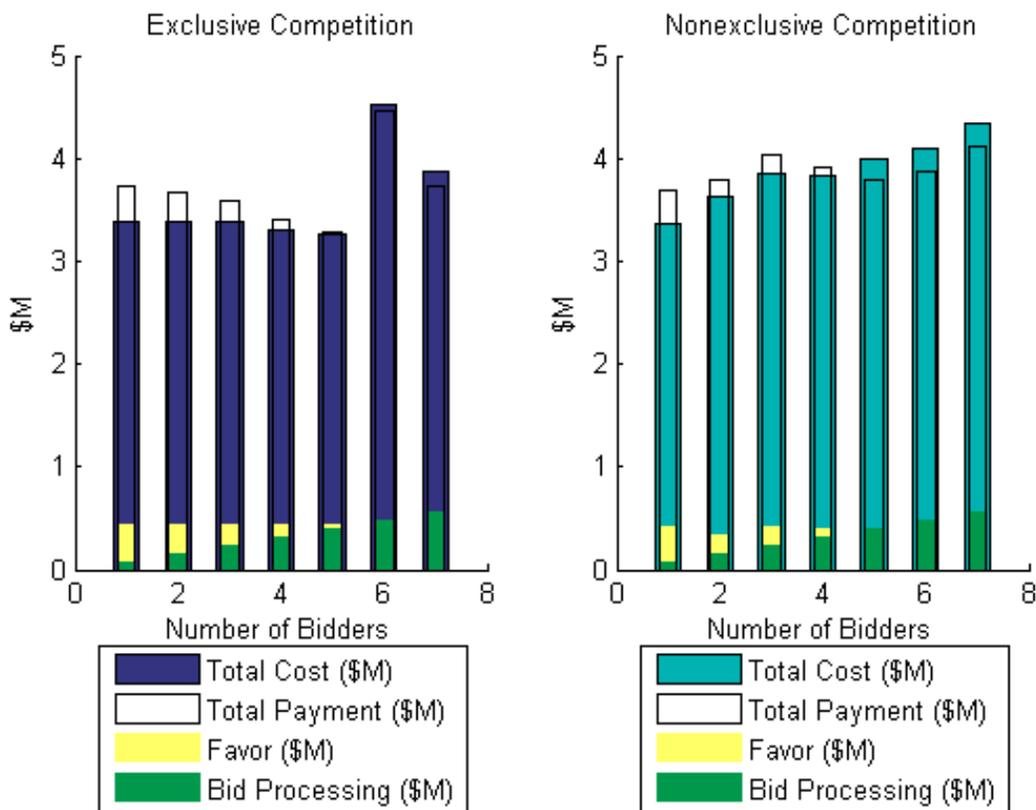
Figure 5: Ratio of Favored Contractors by # of Bidders



6.3 Why is There So Little Competition?

Given our estimates, we calculate the total cost of procurement, which includes (i) the cost of completing a project to the winning contractor, (ii) the bid solicitation processing cost, and (iii) the private benefit of hiring a favored contractor. Table 6 shows how the number of the participants is correlated with the itemized expected cost of procurement with and without eligibility restrictions, respectively.

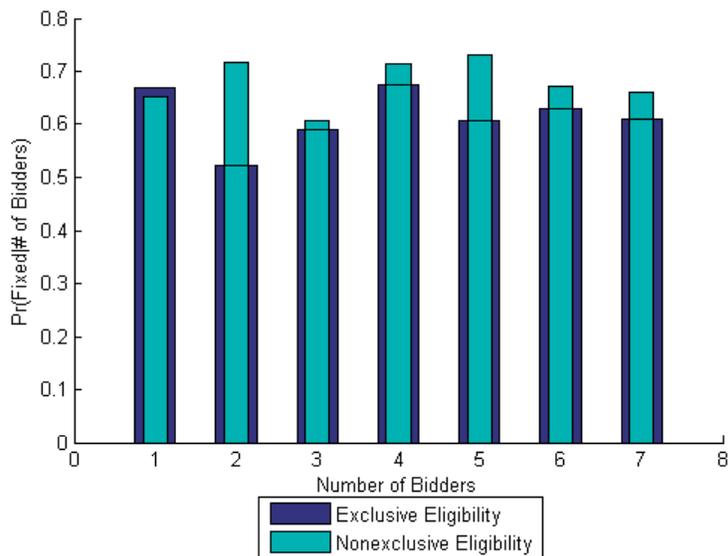
Figure 6: Cost of Procurement by Number of Bidders



We find that the total cost of procurement is, on average, less for exclusive contracts than for nonexclusive contracts. This result may be counter-intuitive, in particular because it is more likely to have fixed-price contracts with eligibility restrictions than without them, as can be seen in Table 7.

Our model and the estimates, however, help us understand this finding. When favoritism exists and bid-processing cost is relatively high, exclusive competition can indeed lead to a lower cost to the government for the following reasons. First, we find that the ratio of efficient contractors decreases as the number of bidders increases. With the relatively large per-bidder bid-processing cost, which is about 18% of the estimated private benefit from favoritism, this

Figure 7: Ratio of Fixed-price Contracts by Number of Bidders

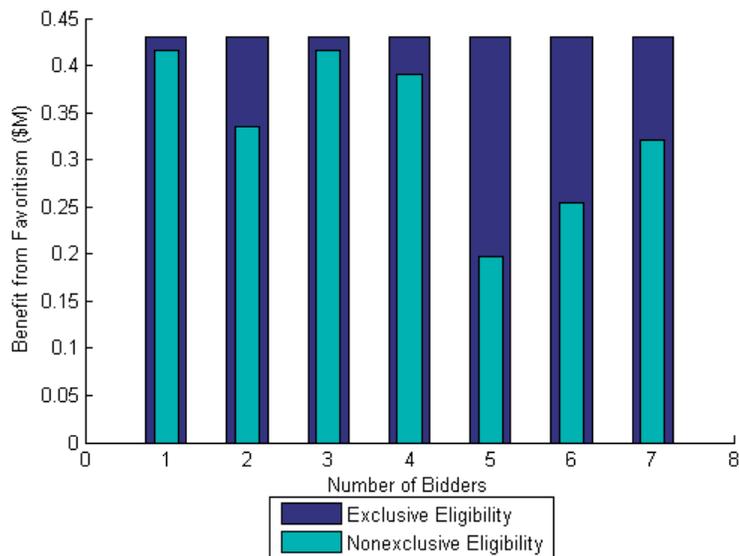


results in a relatively small number of bidders that the procurer intends to seek. Second, given the same number of participants, favored contractors tend to bid less aggressively under nonexclusive competition than under exclusive competition, i.e., the incentive compatibility constraint for efficient, favored contractors is more binding under nonexclusive competition than under exclusive competition. Although favoritism does encourage unfavored contractors to bid more aggressively, the ratio of participating unfavored contractors is not large enough to dominate the opposite effect.

In Table 8, we show that the expected amount of private benefits from hiring a favored contractor under nonexclusive competition decreases as the number of participating bidders increases. This can partially explain the relatively small number of bidders. An interesting finding is that because the favored contractors tend to win when the number of participating bidders is small, the effect of this favoritism on the choice of eligibility rule is not very large.

Note that these findings are confined to the contracts with the median value of the observed characteristics. We will conduct comparative analyses, in particular by varying the size of the contracts and the level of the observed corruption, to further study the effects of the eligibility restrictions on the total cost of procurement.

Figure 8: Benefit from Favoritism by Number of Bidders



7 Conclusion

In this paper, we study the determinants of eligibility requirements and the number of participating bidders in government procurement auctions. We find evidence that less competition is correlated with various measures of political connection and corruption. To understand the effects of the restrictions of competition on the total cost of government procurement, we develop, identify, and estimate a principal-agent model in which the government selects a contractor to undertake a project. We consider three reasons why restricting entry could be beneficial to the government: by decreasing bid processing and solicitation costs, by increasing the chance of selecting a favored contractor and consequently reaping benefits from the favored contractor, and by decreasing the expected amount of payment to the winning contractor. Using our estimates, we decompose the effects of these three sources of entry restrictions, and quantify the effects of the eligibility restrictions on the total cost of procurement.

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Appendix

A. Proofs

A.1. Proof of Theorem 1

We first prove four lemmas that provide several necessary conditions that an equilibrium contract menu must satisfy. We show the equilibrium menu always includes only one fixed price contract by Lemmas 2 and 3. Then we show in Lemma 4 that if inefficient agent maximizes his expected payoff by accepting a fixed-price contract, then it must be that the menu includes only one fixed-price contract. This implies that if there are more than one contract in the equilibrium menu, a fixed-price contract must be accepted by the efficient agents only. In Lemma 5, the efficient agents never accept a variable-price contract in an equilibrium menu. Using the four lemmas, we prove Theorem 1.

The first lemma shows that variable price contracts are only offered in conjunction with fixed price contracts, not by themselves.

Lemma 2 *The equilibrium contract menu includes a fixed price contract.*

Proof. The proof is by a contradiction argument. Suppose to the contrary that every contract on the menu is a variable price contract. Denote by $\{p, q(s)\}$ one of the contracts on the menu. There are three cases to consider.

First, suppose $\mathbb{E}\{\psi[q(s)]|\bar{\beta}\} > \mathbb{E}\{\psi[q(s)]|\underline{\beta}\}$. Then, the procurer can offer an additional, fixed price contract of $p' = p + \mathbb{E}\{\psi[q(s)]|\bar{\beta}\}$. The inefficient agent would accept the contract, but the efficient agent will not. By strict concavity of $\psi(\cdot)$, we have $\mathbb{E}\{\psi[q(s)]|\bar{\beta}\} < \mathbb{E}\{q(s)|\bar{\beta}\}$. Therefore, the expected payoff of the procurer increases when the inefficient agent accept the fixed price contract with any positive probability.

Second, suppose $\mathbb{E}\{\psi[q(s)]|\underline{\beta}\} > \mathbb{E}\{\psi[q(s)]|\bar{\beta}\}$. The procurer can offer an additional, fixed price contract of $p' = p + \mathbb{E}\{\psi[q(s)]|\underline{\beta}\}$. The efficient agent would accept the contract, but the inefficient agent will not. Since $\mathbb{E}\{\psi[q(s)]|\underline{\beta}\} < \mathbb{E}\{q(s)|\underline{\beta}\}$, the expected payoff of the procurer increases when the efficient agent to accept the new contract with any positive probability.

Lastly, suppose $\mathbb{E}\{\psi[q(s)]|\underline{\beta}\} = \mathbb{E}\{\psi[q(s)]|\bar{\beta}\}$. The procurer can offer instead an fixed price contract of $p' = p + \mathbb{E}\{\psi[q(s)]|\underline{\beta}\}$. Both types of agent would accept the contract. Since $\mathbb{E}\{\psi[q(s)]|\underline{\beta}\} < \mathbb{E}\{q(s)|\underline{\beta}\}$ the expected payoff of the procurer increases when either or both agent types to accept the new contract with any positive probability. ■

The next lemma shows that if there are multiple fixed price contracts in the menu, then the procurer can achieve the same expected payoff with only one fixed price contract in

the menu. Furthermore, if there is only one contract in the menu, it must be a fixed-price contract.

Lemma 3 *There is exactly one fixed price contract on the equilibrium menu.*

Proof. Suppose there are multiple contracts in the menu. Among them, if there are multiple, distinct fixed price contracts, then one of them will dominate the rest, and be the only one chosen by the agents.

Suppose there is only one contract in the menu. If the contract is variable contract, then we have shown that in Lemma 2 either offering another fixed-price contract or replacing the variable contract with a fixed-price contract is more profitable to the principal. Therefore, if there is only one contract in the menu, it must be a fixed price contract. ■

The next lemma shows that if it is a best response for the inefficient agent to accept a fixed price contract in equilibrium, then that is the only contract offered on the equilibrium menu.

Lemma 4 *If a fixed price contract is on the equilibrium menu, and the inefficient agent would maximize his expected payoff by accepting it, then the only contract on the menu is the fixed price contract $c + \bar{\beta}$.*

Proof. Let p denote the fixed price contract in question. Suppose there is another contract in the menu. By Lemma 3, that contract is a variable contract. denoted by $\{p', q(\cdot)\}$.

Suppose this variable contract is equally attractive to the inefficient agent as the fixed price contract. Then, $p' + \mathbb{E}\{\psi[q(s)]|\bar{\beta}\} = p$. Since $\mathbb{E}\{\psi[q(s)]|\bar{\beta}\} < \mathbb{E}[q(s)|\bar{\beta}]$, the procurer earns less rent than what would be obtained from the inefficient agent accepting the fixed price contract.

Now that the variable contract is not chosen by the inefficient agent, it must be chosen by the efficient agent with any positive probability. This requires that $p' + \mathbb{E}\{\psi[q(s)]|\underline{\beta}\} \geq p$. Since $\mathbb{E}\{\psi[q(s)]|\underline{\beta}\} < \mathbb{E}[q(s)|\underline{\beta}]$, the procurer earns less than the rent from the efficient agent with the fixed price. This proves that if a fixed-price contract is acceptable to the inefficient agent, the procurer does not offer a variable-price contract in equilibrium. ■

By Lemma 3, there is exactly one fixed-price contract on the equilibrium menu. If there is more than one contract on the equilibrium menu, by Lemma 4, the fixed price contracts are only accepted by the efficient agent. The next result, Lemma 5, establishes that in equilibrium the efficient agent never accepts a variable-price contract.

Lemma 5 *The efficient agent accepts the fixed price contract on the equilibrium menu with unit probability.*

Proof. By Lemma 4, the efficient agent would maximize his payoff by accepting the fixed-price contract, which we denote by p . So if a variable-price contract of the form $\{p', q(\cdot)\}$ is offered, incentive compatibility requires: $p \geq p' + \mathbb{E} \{ \psi [q(s)] | \underline{\beta} \}$. If a strict inequality holds for all variable contracts on the menu, the result follows immediately. Supposing an equality holds for a variable-price contract, then the procurer garners $\mathbb{E} \{ q(s) - \psi [q(s)] | \underline{\beta} \}$ less rent if the efficient agent selects the variable-price contract than the fixed-price contract. Therefore, in equilibrium the principal does not offer a variable contract that an efficient agent would accept with any positive probability. ■

We conclude that in equilibrium either there is one fixed price or there are several contracts in the menu, which includes one fixed price contract that the efficient agents select into, and one or more variable contracts that the inefficient agents select into. Using the strict convexity of the constraint set we show that only one variable price contract will be offered.

Proof. We consider the following sub-problem of the procurer. The procurer minimizes the expected cost of attracting the inefficient agent subject to the constraint that the efficient agent is deterred from taking the contract given an outside option that provides an exogenously determined payoff. The IR constraint for the inefficient agent is:

$$\bar{p} + \mathbb{E} \{ \psi [\bar{q}(s)] | \bar{\beta} \} - (c + \bar{\beta}) \geq 0,$$

and the IC constraint for the efficient agent is:

$$\underline{p} \geq \bar{p} + \mathbb{E} \{ \psi [\bar{q}(s)] | \underline{\beta} \}.$$

By inspection the IR constraint holds with equality. (If the IR constraint is not binding, then the value of objective function can be reduced by reducing \bar{p} .) Solving for \bar{p} reduces the problem to choosing $\bar{q}(s)$ to minimize:

$$c + \bar{\beta} + \mathbb{E} [\bar{q}(s) | \bar{\beta}] - \mathbb{E} \{ \psi [\bar{q}(s)] | \bar{\beta} \}$$

subject to the constraint:

$$\underline{p} \geq c + \bar{\beta} - \mathbb{E} \{ \psi [\bar{q}(s)] | \bar{\beta} \} + \mathbb{E} \{ \psi [\bar{q}(s)] | \underline{\beta} \}.$$

There are two cases to consider. First suppose the constraint is not binding. Since $\psi(\cdot)$ is concave, the objective function is convex, so the unconstrained problem has a unique

solution, characterized by the first order condition $\psi' [\bar{q}(s)] = 1$ for all s , implying $\bar{q}(s) = 0$. Substituting the unconstrained solution $\bar{p} = c + \bar{\beta}$ into the IC constraint for the efficient agent it follows that $\underline{p} \geq c + \bar{\beta}$ in this case. Otherwise the constraint is binding.

Alternatively assume the constraint is met with equality. Then, the above constrained minimization problem can be written to be an unconstrained minimization problem with the objective function

$$\underline{p} + \mathbb{E} [\bar{q}(s) | \bar{\beta}] - \mathbb{E} \{ \psi [\bar{q}(s)] | \underline{\beta} \}$$

Since $\psi(\cdot)$ is concave, the objective function is convex, so the above problem also has a unique solution characterized by its first order condition $\psi' [\bar{q}(s)] f(s | \underline{\beta}) = f(s | \bar{\beta})$. Since the solution to the subproblem is unique, all the variable contracts solving the subproblems are identical, thus proving only one (distinct) variable contract is offered to the inefficient agent. ■

A.2. Proof of Theorem 2

Proof. In Theorem 1, we have established that necessary conditions for equilibrium that include: (i) there are at most two contracts offered by the procurer; (ii) one of the contracts is fixed-price; (iii) when two contracts are offered, the other is variable-price; and (iv) when two contracts are offered, the inefficient agent accepts the variable price contract and the efficient agent accepts the fixed price contract. These conditions provide the basis for the setup in the text when two contracts are offered in the menu. There are incentive compatibility (IC) and individual rationality (IR) constraints for each agent type, and the procurer minimizes:

$$\pi \underline{p} + (1 - \pi) \{ \bar{p} + \mathbb{E} [\bar{q}(s) | \bar{\beta}] \}$$

subject to the IC and IR constraints for the efficient agent and the IR constraint for the inefficient agent:

$$\underline{p} \geq \bar{p} + \mathbb{E} \{ \psi [\bar{q}(s)] | \underline{\beta} \},$$

$$\underline{p} \geq c + \underline{\beta},$$

$$\bar{p} + \mathbb{E} \{ \psi [\bar{q}(s)] | \bar{\beta} \} \geq c + \bar{\beta}.$$

We ignore the IC constraint for the inefficient type because by Lemma 4, the solution of the above problem satisfies the constraint.

Note that if the IR constraint for the inefficient type is not binding, then reducing \bar{p} further weakens the IC constraint for the efficient agent and also reduces the procurer's expected cost. Hence a necessary condition for optimization is that the IR constraint of the

inefficient agent binds. Solving for \bar{p} from the IR constraint equality gives

$$\bar{p} = c + \bar{\beta} - \mathbb{E} \{ \psi [\bar{q}(s)] | \bar{\beta} \},$$

and substituting the solution into the objective function, which yields:

$$\pi \underline{p} + (1 - \pi) (c + \bar{\beta} - \mathbb{E} \{ \psi [\bar{q}(s)] | \bar{\beta} \} + \mathbb{E} [\bar{q}(s) | \bar{\beta}]),$$

and also into the IC constraint to obtain:

$$\underline{p} \geq c + \bar{\beta} - \mathbb{E} \{ \psi [\bar{q}(s)] | \bar{\beta} \} + \mathbb{E} \{ \psi [\bar{q}(s)] | \underline{\beta} \}.$$

Note that if neither IC constraint nor IR constraint for the efficient type binds, then \underline{p} can be reduced without violating either constraint, thus reducing the value of the procurer's objective function. Consequently at least one of the constraints binds at the optimum. There are three logical possibilities, depending on which of the remaining constraints bind at the optimum.

First, suppose that the IC constraint for the efficient agent does not bind at the optimum. Then $\underline{p} = c + \underline{\beta}$ (since at least one constraint binds), and we can substitute for \underline{p} in the objective function, which becomes:

$$\pi(c + \underline{\beta}) + (1 - \pi) (c + \bar{\beta} - \mathbb{E} \{ \psi [\bar{q}(s)] | \bar{\beta} \} + \mathbb{E} [\bar{q}(s) | \bar{\beta}]).$$

Since $\mathbb{E} \{ \psi [q(s)] | \bar{\beta} \} < \mathbb{E} [q(s) | \bar{\beta}]$ for all $q(s) \neq 0$, the solution to the optimization problem above is to set $q(s) = 0$ and (from above) $\bar{p} = c + \bar{\beta}$. However, this contradicts Lemma 3 that there is only one fixed price on the equilibrium menu because $\underline{\beta} \neq \bar{\beta}$. Therefore the IC constraint for the efficient agent binds.

Since the IC constraint for the efficient agent binds,

$$\underline{p} = c + \bar{\beta} - \mathbb{E} \{ \psi [\bar{q}(s)] | \bar{\beta} \} + \mathbb{E} \{ \psi [\bar{q}(s)] | \underline{\beta} \}.$$

Substituting this equality into the procurer's modified objective function and the remaining IR constraint for the efficient agent, the optimization problem is to choose $\bar{q}(\cdot)$ to minimize:

$$c + \bar{\beta} - \mathbb{E} \{ \psi [\bar{q}(s)] | \bar{\beta} \} + \pi \mathbb{E} \{ \psi [\bar{q}(s)] | \underline{\beta} \} + (1 - \pi) \mathbb{E} [\bar{q}(s) | \bar{\beta}], \quad (14)$$

subject to the IR constraint for the efficient agent now rewritten as:

$$\bar{\beta} - \underline{\beta} \geq \mathbb{E} \{ \psi [\bar{q}(s)] | \bar{\beta} \} - \mathbb{E} \{ \psi [\bar{q}(s)] | \underline{\beta} \}.$$

We now show that the IR constraint for the efficient type does not bind at the optimum by contradiction. Suppose the IR constraint is binding at the optimum. Then:

$$\mathbb{E} \{ \psi [\bar{q}(s)] | \underline{\beta} \} = \mathbb{E} \{ \psi [\bar{q}(s)] | \bar{\beta} \} - \bar{\beta} + \underline{\beta}.$$

Substituting for $\mathbb{E} \{ \psi [\bar{q}(s)] | \underline{\beta} \}$ in the objective function (14) gives:

$$c + \pi \underline{\beta} + (1 - \pi) (\bar{\beta} + \mathbb{E} [\bar{q}(s) | \bar{\beta}] - \mathbb{E} \{ \psi [\bar{q}(s)] | \bar{\beta} \}).$$

Note the resulting function is strictly convex in \bar{q} . Differentiating we obtain the first order condition that $\psi' [\bar{q}(s)] = 1$, uniquely solved by $\bar{q}(s) = 0$. This contradicts Lemma 3. This proves that the IR constraint for the efficient agent never binds.

We have proved that the IC constraint for the efficient agent and the IR constraint for the inefficient agent are binding at the optimum. Substituting the equalities from the binding constraints into the procurer's objective function, we obtain (14). By taking the first derivative with respect to $\bar{q}(s)$,

$$\psi' [\bar{q}(s)] \{ -f(s|\bar{\beta}) + \pi f(s|\underline{\beta}) \} + (1 - \pi) f(s|\bar{\beta}).$$

Notice that if $-f(s|\bar{\beta}) + \pi f(s|\underline{\beta}) \geq 0$, then the above expression is positive. Therefore, if $f(s|\underline{\beta})/f(s|\bar{\beta}) \geq 1/v$, then $\bar{q}(s)$ is not an interior solution. If there exists a maximal penalty that the procurer can legally impose on contractors, denoted by $M < 0$, $\bar{q}(s) = M$. If $f(s|\underline{\beta})/f(s|\bar{\beta}) < 1/v$, then $\bar{q}(s)$ is an interior solution. ■

A.3. Proof of Theorem 3

Proof. Suppose such S does not exist. This implies that there exists no $\eta > 0$ such that $S \equiv \{s : f(s|\bar{\beta}) - f(s|\underline{\beta}) > \eta\}$. It is because by construction, $\gamma_1 < \gamma_2$. Therefore, $\Pr \{s : f(s|\underline{\beta}) = f(s|\bar{\beta})\} = 1$. For any ex-post price adjustment $\bar{q}(\cdot)$, $\mathbb{E}(\psi(\bar{q})|\underline{\beta}) = \mathbb{E}(\psi(\bar{q})|\bar{\beta})$. Hence the individual rationality constraint for the inefficient agent implies

$$\begin{aligned} c + \bar{\beta} &\leq \bar{p} + \mathbb{E}(\psi(\bar{q})|\bar{\beta}) \\ &= \bar{p} + \mathbb{E}(\psi(\bar{q})|\underline{\beta}) \leq \underline{p}. \end{aligned}$$

The last inequality holds due to the incentive compatibility constraint for the efficient agent. Therefore the efficient agent extracts rent of at least $\bar{\beta}$. The pooling equilibrium is more efficient than inducing the inefficient agent to take a variable contract because the procurer does not pay a liquidity premium of $\mathbb{E}(\psi(\bar{q})|\bar{\beta})$ to the inefficient agent in the pooling case.

Now suppose S exists. In that case, we show by construction that it is less profitable to offer one fixed-price contract than a menu of two contracts, one fixed-price and the other variable-price. For any $\Delta > 0$ choose $\mu(\Delta)$ for a two-part variable contract in which $\bar{p} = c + \bar{\beta}$ and:

$$\bar{q}(s) = \begin{cases} \Delta & \text{if } s \in S, \\ \mu(\Delta) & \text{if } s \notin S, \end{cases}$$

where

$$\gamma_2 \psi(\Delta) + (1 - \gamma_2) \psi(\mu(\Delta)) = 0.$$

Note that the above equation implies that $\mu(\Delta) < 0$. Because $\psi(\cdot)$ is strictly increasing, $\mu(\Delta)$ is uniquely defined by the equation:

$$\mu(\Delta) = \psi^{-1} \left[\frac{-\gamma_2}{1 - \gamma_2} \psi(\Delta) \right],$$

and is twice differentiable with:

$$\mu'(\Delta) = \frac{-\gamma_2}{1 - \gamma_2} \frac{\psi'(\Delta)}{\psi'(\mu(\Delta))},$$

where $\mu(0) = 0$. The fixed contract takes the form:

$$\begin{aligned} \bar{p} &= c + \bar{\beta} + \gamma_1 \psi(\Delta) + (1 - \gamma_1) \psi(\mu(\Delta)) \\ &= c + \bar{\beta} + \gamma_1 \psi(\Delta) - (1 - \kappa_1) \left(\frac{\gamma_2}{1 - \kappa_2} \right) \psi(\Delta). \end{aligned}$$

Note that the incentive compatibility constraint is satisfied with equality by the efficient agent and strict inequality by the inefficient agent because $\gamma_1 < \gamma_2$. Similarly, the participation constraint is satisfied with equality by the inefficient agent and strict inequality by the efficient agent as long as $\Delta > 0$ is small enough. The expected payment by the procurer is:

$$\begin{aligned} \mathbb{E}(T|\Delta) &= c + \bar{\beta} + \pi [\gamma_1 \psi(\Delta) + (1 - \gamma_1) \psi(\mu(\Delta))] + (1 - \pi) [\gamma_2 \Delta + (1 - \gamma_2) \mu(\Delta)], \\ &= c + \bar{\beta} + \pi \left[\kappa_1 \psi(\Delta) - \frac{(1 - \gamma_1) \gamma_2}{1 - \gamma_2} \psi(\Delta) \right] + (1 - \pi) [\gamma_2 \Delta + (1 - \gamma_2) \mu(\Delta)]. \end{aligned}$$

We now show this expression is decreasing in the neighborhood of $\Delta = 0$. Differentiating

with respect to Δ yields:

$$\frac{\partial \mathbb{E}(T|\Delta)}{\partial \Delta} = \pi \left[\gamma_1 \psi'(\Delta) - \frac{(1 - \gamma_1) \gamma_2}{1 - \gamma_2} \psi'(\Delta) \right] + (1 - \pi) \left[\gamma_2 - \gamma_2 \frac{\psi'(\Delta)}{\psi'(\mu(\Delta))} \right].$$

Evaluating $\frac{\partial \mathbb{E}(T|\Delta)}{\partial \Delta}$ at $\Delta = 0$ gives us:

$$\frac{\partial \mathbb{E}(T|\Delta = 0)}{\partial \Delta} = \pi \frac{\gamma_1 - \gamma_2}{1 - \gamma_2} < 0,$$

which shows that a fixed-price contract fails to meet a first order necessary condition. ■

A.4. Proof of Theorem 4

Proof. The procurer minimizes the expected payment to a winning contractor given n favored participants:

$$\mathbf{E}(T|n, 0) = \Pr(\text{fixed}|n, 0) \underline{p} + (1 - \Pr(\text{fixed}|n, 0)) [\bar{p} + \mathbf{E}(\bar{q}(s)|\bar{\beta})],$$

where the probability that the winning contractor is efficient (i.e., the winning contract is fixed-price), denoted by $\Pr(\text{fixed}|n, 0)$, is

$$\Pr(\text{fixed}|n, 0) = 1 - (1 - \pi_f(n))^n.$$

Given that the incentive constraint for the efficient contractors and the participation constraint for the inefficient contractors are binding at the optimum, we have the following two equations:

$$\begin{aligned} \underline{\phi}_f(n) \{ \underline{p} - (c + \underline{\beta}) \} &= \bar{\phi}_f(n) \{ \bar{p} + \mathbf{E}(\psi(q)|\underline{\beta}) - (c + \underline{\beta}) \}, \\ \bar{p} &= -\mathbf{E}(\psi(q)|\bar{\beta}) + (c + \bar{\beta}), \end{aligned}$$

Rearranging the terms, we have

$$\begin{aligned} \underline{p} &= c + \underline{\beta} - \frac{\bar{\phi}_f(n)}{\underline{\phi}_f(n)} \left[\underline{\beta} - \bar{\beta} + \int \psi(\bar{q}(s)) [f(s|\bar{\beta}) - f(s|\underline{\beta})] ds \right], \\ \bar{p} &= c + \bar{\beta} - \int \psi(\bar{q}(s)) f(s|\bar{\beta}) ds. \end{aligned}$$

We substitute the above equations for \underline{p} and \bar{p} in the expected payment of the procurer, resulting in the following:

$$\begin{aligned} \mathbf{E}(T|n, 0) &= \Pr(\text{fixed}|n, 0) \left[c + \underline{\beta} - \frac{\bar{\phi}_f(n)}{\underline{\phi}_f(n)} \{ \underline{\beta} - \bar{\beta} + \mathbb{E}(\psi(\bar{q}(s))|\bar{\beta}) - \mathbb{E}(\psi(\bar{q}(s))|\underline{\beta}) \} \right] \\ &+ (1 - \Pr(\text{fixed}|n, 0)) [c + \bar{\beta} - \mathbb{E}(\psi(\bar{q})|\bar{\beta}) + \mathbb{E}(\bar{q}(s)|\bar{\beta})]. \end{aligned}$$

If s is such that

$$\frac{f(s|\underline{\beta})}{f(s|\bar{\beta})} < 1 + \frac{n(1 - \pi_f(n))\underline{\phi}_f(n)}{1 - n(1 - \pi_f(n))\bar{\phi}_f(n)},$$

then the optimal $\bar{q}(s)$ satisfies the following first order condition:

$$\begin{aligned} &\Pr(\text{fixed}|n, 0) \frac{\bar{\phi}_f(n)}{\underline{\phi}_f(n)} \psi'(\bar{q}(s)) [-f(s|\bar{\beta}) + f(s|\underline{\beta})] \\ &+ (1 - \Pr(\text{fixed}|n, 0)) [-\psi'(\bar{q}(s))f(s|\bar{\beta}) + f(s|\bar{\beta})] = 0. \end{aligned}$$

Rearranging terms after dividing both sides of the above equation by $f(s|\bar{\beta})(1 - \Pr(\text{fixed}|n, 0))$ gives us:

$$\psi'(\bar{q}(s)) \left(1 + \frac{\Pr(\text{fixed}|n, 0)}{1 - \Pr(\text{fixed}|n, 0)} \frac{\bar{\phi}_f(n)}{\underline{\phi}_f(n)} \left[1 - \frac{f(s|\underline{\beta})}{f(s|\bar{\beta})} \right] \right) = 1.$$

Note that $\Pr(\text{fixed}|n, 0)$ and $\bar{\phi}_f(n)$ have the following relationship:

$$\Pr(\text{fixed}|n, 0) = 1 - n(1 - \pi_f(n))\bar{\phi}_f(n).$$

Plugging the above equation in, we have the following characterization of $\bar{q}(s)$:

$$\psi'(\bar{q}(s)) \left(1 + \frac{1 - n(1 - \pi_f(n))\bar{\phi}_f(n)}{n(1 - \pi_f(n))\underline{\phi}_f(n)} \left[1 - \frac{f(s|\underline{\beta})}{f(s|\bar{\beta})} \right] \right) = 1.$$

If s is such that

$$\frac{f(s|\underline{\beta})}{f(s|\bar{\beta})} \geq 1 + \frac{n(1 - \pi_f(n))\underline{\phi}_f(n)}{1 - n(1 - \pi_f(n))\bar{\phi}_f(n)},$$

then the expected payment, $\mathbf{E}(T|n, 0)$, is increasing in $\bar{q}(s)$. Therefore, the optimal $\bar{q}(s)$ must take the smallest payment that the government can legally give, i.e., M . ■

A.5. Proof of Theorem 5

Proof. The procurer minimizes the expected payment to a winning contractor given n_f favored participants and n_u unfavored participants:

$$\begin{aligned} \mathbf{E}(T|n_f, n_u) &= \Pr(\text{fixed, favored}|n_f, n_u)\underline{p} + \Pr(\text{variable, favored}|n_f, n_u) [\bar{p} + \mathbf{E}(\bar{q}(s)|\bar{\beta})] \\ &+ \Pr(\text{unfavored}|n_f, n_u)c, \end{aligned}$$

where the probability that the winning contractor is efficient (i.e., the winning contract is fixed-price) and favored is

$$\Pr(\text{fixed, favored}|n_f, n_u) = 1 - (1 - \pi_f(n_f))^{n_f},$$

and the probability that the winning contractor is inefficient and favored is

$$\Pr(\text{variable, favored}|n_f, n_u) = (1 - \pi_f(n_f))^{n_f} (1 - \pi_u(n_u))^{n_u}.$$

Given that the incentive constraint for the efficient contractors and the participation constraint for the inefficient contractors are binding at the optimum, we have the following two equations:

$$\underline{\phi}(n_f)\{\underline{p} - (c + \underline{\beta})\} = \bar{\phi}(n_f, n_u)\{\bar{p} + \mathbf{E}(\psi(q)|\bar{\beta}) - (c + \underline{\beta})\},$$

$$\bar{p} = c + \bar{\beta} - \mathbf{E}[\psi(\bar{q}(s))|\bar{\beta}],$$

Rearranging terms, we have

$$\underline{p} = c + \underline{\beta} - \frac{\bar{\phi}(n_f, n_u)}{\underline{\phi}(n_f)} \left[\underline{\beta} - \bar{\beta} + \int \psi(\bar{q}(s)) [f(s|\bar{\beta}) - f(s|\underline{\beta})] ds \right], \quad (15)$$

$$\bar{p} = c + \bar{\beta} - \int \psi(\bar{q}(s)) f(s|\bar{\beta}) ds, \quad (16)$$

where $\bar{\phi}(n_f)$ (or $\underline{\phi}(n_f, n_u)$) denotes the probability of winning for a favored contractor if he accepts a fixed-price (or variable-price) contract.

$$\begin{aligned} \underline{\phi}(n_f) &= \sum_{k=0}^{n_f-1} \binom{n_f-1}{k} \frac{\pi_f(n_f)^k (1 - \pi_f(n_f))^{n_f-1-k}}{k+1}, \\ \bar{\phi}(n_f, n_u) &= \frac{(1 - \pi_f(n_f))^{n_f-1} (1 - \pi_u(n_u))^{n_u}}{n_f}. \end{aligned}$$

Similarly in the proof for Theorem 4, the optimal $\bar{q}(s)$ satisfies the following first order condition:

$$\begin{aligned} & \Pr(\text{fixed, favored}|n_f, n_u) \frac{\bar{\phi}(n_f, n_u)}{\underline{\phi}(n_f, n_u)} \psi'(\bar{q}(s)) [-f(s|\bar{\beta}) + f(s|\underline{\beta})] \\ & + \Pr(\text{variable, favored}|n_f, n_u) [-\psi'(\bar{q}(s))f(s|\bar{\beta}) + f(s|\bar{\beta})] = 0. \end{aligned}$$

Rearranging terms after dividing both sides of the above equation by $f(s|\bar{\beta}) \Pr(\text{var.}, \text{fav.}|n_f, n_u)$ gives us:

$$\psi'(\bar{q}(s)) \left(1 + \frac{\Pr(\text{fixed, favored}|n_f, n_u) \bar{\phi}(n_f, n_u)}{\Pr(\text{variable, favored}|n_f, n_u) \underline{\phi}(n_f, n_u)} \left[1 - \frac{f(s|\underline{\beta})}{f(s|\bar{\beta})} \right] \right) = 1. \quad (17)$$

If s is such that the above first order condition does not hold, the optimal $\bar{q}(s)$ must take the smallest payment that the government can legally give, i.e., M . ■

A.6. Proof of Proposition 1

Proof. The ex-post payment adjustment for the variable-price contracts won in a nonexclusive competition with a single bidder when a signal s is realized is the weighted average of two payment adjustment schedules, \bar{q}_f for a favored contractor and \bar{q}_u for an unfavored contractor. For notational easiness, let ρ denote $\rho(1)$ in the proof.

$$[1 - \Pr(\text{fixed}|nonexc., 1)]\bar{q}(s) = \rho(1 - \pi_f(1))\bar{q}_f(s) + (1 - \rho)(1 - \pi_u(1; \rho))\bar{q}_u(s; \rho), \quad (18)$$

where $\bar{q}_u(s; \rho)$ is defined as:

$$\bar{q}_u(s; \rho) = \begin{cases} \psi'^{-1} \left(\frac{1 - \pi_u(1, \rho)}{1 - \pi_u(1, \rho) \frac{f(s|\underline{\beta})}{f(s|\bar{\beta})}} \right) & \text{if } \frac{f(s|\underline{\beta})}{f(s|\bar{\beta})} < \frac{1}{\pi_u(1; \rho)}, \\ M & \text{otherwise.} \end{cases}$$

Here, the probability that an unfavored single participant is efficient, $\pi_u(1; \rho)$, is defined as:

$$\pi_u(1; \rho) = \frac{\Pr(\text{fixed}|nonexc., 1) - \rho\pi_f(1)}{1 - \rho}.$$

Note that $\bar{q}(s)$, $\bar{q}_f(s)$, $f(s|\beta)$, $\pi_f(1)$, and ψ'^{-1} have been identified. Therefore, the proof is done if there exists a set of s with nonzero measure that equation (18) holds for a unique ρ .

First, consider the case when $\pi_f(1) > \Pr(\text{fixed}|nonexclusive, 1)$. This implies that $\pi_u(1)$

is less than $\pi_f(1)$. We show that the right hand side of equation (18) is strictly increasing in ρ for any given s such that $1 < f(s|\underline{\beta})/f(s|\bar{\beta}) < 1/\pi_f(1)$. Because the support of s is convex, the measure of such s is positive. The first derivative of the right hand side of equation (18) is:

$$\left\{ (1 - \pi_f(1))\bar{q}_f(s) - \left[1 - \pi_u(1; \rho) + (1 - \rho) \frac{\partial \pi_u(s; \rho)}{\partial \rho} \right] \bar{q}_u(s; \rho) \right\} + (1 - \rho)(1 - \pi_u(1; \rho)) \frac{\partial \bar{q}_u(s; \rho)}{\partial \rho}. \quad (19)$$

To sign the first term of (19), we compare $\bar{q}_f(s)$ and $\bar{q}_u(s; \rho)$. Because $\pi_u(1)$ is less than $\pi_f(1)$ and $f(s|\underline{\beta})/f(s|\bar{\beta}) < 1/\pi_f(1)$, $\bar{q}_f(s)$ and $\bar{q}_u(s; \rho)$ can be represented as follows:

$$\bar{q}_f(s) = \psi'^{-1} \left(\frac{1 - \pi_f(1)}{1 - \pi_f(1) \frac{f(s|\underline{\beta})}{f(s|\bar{\beta})}} \right),$$

$$\bar{q}_u(s; \rho) = \psi'^{-1} \left(\frac{1 - \pi_u(1; \rho)}{1 - \pi_u(1; \rho) \frac{f(s|\underline{\beta})}{f(s|\bar{\beta})}} \right).$$

Given that (i) ψ is strictly concave, (ii) $\pi_u(1, \rho)$ is less than $\pi_f(1)$ for any $\rho \in (0, 1)$, and (iii) $f(s|\underline{\beta})/f(s|\bar{\beta}) > 1$, it can be seen that $\bar{q}_f(s) - \bar{q}_u(s; \rho) > 0$.

Then we compare the coefficients for $\bar{q}_f(s)$ and $\bar{q}_u(s; \rho)$ in the first term of (19).

$$(1 - \pi_f(1)) - \left[1 - \pi_u(1; \rho) + (1 - \rho) \frac{\partial \pi_u(s; \rho)}{\partial \rho} \right] = \pi_u(1; \rho) - \pi_f(1) - (1 - \rho) \frac{\partial \pi_u(s; \rho)}{\partial \rho}$$

$$= \pi_u(1; \rho) - \pi_f(1) + \frac{\Pr(\text{fixed}|\text{nonexc.}, 1) + \pi_f(1) - 2\rho\pi_f(1)}{1 - \rho} = 2\pi_u(1; \rho) > 0.$$

The last equality is derived from the relationship between $\Pr(\text{fixed}|\text{nonexclusive}, 1)$ and $(\pi_u(1), \pi_f(1), \rho)$. Because (i) the coefficient for $\bar{q}_f(s)$ is larger than that of $\bar{q}_u(s; \rho)$, and (ii) $\bar{q}_f(s) - \bar{q}_u(s; \rho) > 0$, we conclude that the first term of (19) is positive.

To sign the second term of (19), we expand the expression for $\partial \bar{q}_u(s, \rho) / \partial \rho$:

$$\frac{\partial \bar{q}_u(s, \rho)}{\partial \rho} = (\psi'^{-1})' \left(\frac{1 - \pi_u(1; \rho)}{1 - \pi_u(1; \rho) \frac{f(s|\underline{\beta})}{f(s|\bar{\beta})}} \right) \times \frac{\frac{f(s|\underline{\beta})}{f(s|\bar{\beta})} - 1}{\left(1 - \pi_u(1; \rho) \frac{f(s|\underline{\beta})}{f(s|\bar{\beta})} \right)^2} \times \frac{\Pr(\text{fixed}|\text{nonexc.}, 1) - \pi_f(1)}{(1 - \rho)^2}.$$

Because ψ is strictly concave, the first term in the above equation is negative. $f(s|\underline{\beta})/f(s|\bar{\beta}) > 1$ implies that the second term is positive. The last term is negative because $\pi_f(1) > \Pr(\text{fixed}|\text{nonexc.}, 1)$. Therefore, $\partial \bar{q}_u(s, \rho) / \partial \rho > 0$.

Second, consider the opposite case where $\pi_f(1) < \Pr(\text{fixed}|\text{nonexclusive}, 1)$. This implies

that $\pi_u(1)$ is greater than $\pi_f(1)$. Following a very similar logic for the first case, we can show that the right hand side of equation (18) is strictly increasing in ρ for any given s such that $f(s|\underline{\beta})/f(s|\bar{\beta}) < 1$. Because the support of s is convex, the measure of such s is positive. ■

B. Simulation Procedure

We consider the following components of the model, conditional on the observed characteristics: (i) the distribution of the minimum cost, $G(c)$, represented by two vectors of size n_c ($\{c_1, \dots, c_{n_c}\}$ and $\{g(c_1), \dots, g(c_{n_c})\}$), (ii) the liquidity cost function, $\psi(\cdot)$, (iii) the conditional distribution of signals, $f(\cdot|\underline{\beta})$ and $f(\cdot|\bar{\beta})$, represented by three vectors of size n_s ($\{s_1, \dots, s_{n_s}\}$, $\{f(s_1|\underline{\beta}), \dots, f(s_{n_s}|\underline{\beta})\}$, and $\{f(s_1|\bar{\beta}), \dots, f(s_{n_s}|\bar{\beta})\}$), (iv) the probability that a bidder is efficient if n bidders participate, $\{\pi_f(1), \dots, \pi_f(n_b)\}$ for favored bidders and $\{\pi_u(1), \dots, \pi_u(n_b)\}$ for unfavored ones for $n = 1, \dots, n_b$, (v) the probability that a bidder is favored if n bidders participate ($\{\rho(1), \dots, \rho(n_b)\}$), and (vi) the remaining parameters, i.e., κ , δ , and $\bar{\beta}$.

Equilibrium of the Model Given these components of the model, we solve for the equilibrium of the model: (i) the menus of contracts, (ii) the rates of bidder arrivals for exclusive and non-exclusive competition, and (iii) the decision rule on whether to hold an exclusive competition. The following procedure describes how to solve the equilibrium. First, for exclusively competed contracts, we calculate $\bar{q}(s_i; n)$ for each s_i and n using equation (7), as below:

$$\bar{q}(s_i; n) = \psi'^{-1} \left[\left(1 + \frac{1 - n(1 - \pi_f(n))\bar{\phi}_f(n)}{n(1 - \pi_f(n))\underline{\phi}_f(n)} \left[1 - \frac{f(s_i|\underline{\beta})}{f(s_i|\bar{\beta})} \right] \right)^{-1} \right],$$

where

$$\underline{\phi}_f(n) = \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{\pi_f(n)^k (1 - \pi_f(n))^{n-1-k}}{k+1},$$

$$\bar{\phi}_f(n) = \frac{(1 - \pi_f(n))^{n-1}}{n}.$$

Given $\bar{q}(s_i; n)$'s and equations (5) and (6), we calculate the base payments, $\bar{p}(c_j; n)$ and $\underline{p}(c_j; n)$ as follows:

$$\underline{p}(c_j; n) = c_j + \frac{\bar{\phi}_f(n)}{\underline{\phi}_f(n)} \left[\bar{\beta} - \sum_{i=1}^{n_s} \psi(\bar{q}(s_i; n)) [f(s_i|\bar{\beta}) - f(s_i|\underline{\beta})] |s_i - s_{i-1}| \right].$$

$$\bar{p}(c_j; n) = c_j + \bar{\beta} - \sum_{i=1}^{n_s} \psi(\bar{q}(s_i; n)) f(s_i|\bar{\beta}) |s_i - s_{i-1}|,$$

Second, as for non-exclusively competed contracts for favored contractors, we calculate $\bar{q}(s_i; n_f, n_u)$ for each s_i and (n_f, n_u) using equation (17) as follows:

$$\bar{q}(s_i; n_f, n_u) = \psi'^{-1} \left[\left(1 + \frac{1 - (1 - \pi_f(n_f))^{n_f}}{(1 - \pi_f(n_f))^{n_f} (1 - \pi_u(n_u))^{n_u}} \frac{\bar{\phi}(n_f, n_u)}{\underline{\phi}(n_f)} \left[1 - \frac{f(s_i|\underline{\beta})}{f(s_i|\bar{\beta})} \right] \right)^{-1} \right],$$

where

$$\bar{\phi}(n_f, n_u) = \frac{(1 - \pi_f(n_f))^{n_f-1} (1 - \pi_u(n_u))^{n_u}}{n_f}.$$

We then calculate the base payments, $\bar{p}(c_j, n_f, n_u)$ and $\underline{p}(c_j, n_f, n_u)$, using equations (15) and (16).

$$\underline{p}(c_j, n_f, n_u) = c_j + \frac{\bar{\phi}(n_f, n_u)}{\underline{\phi}(n_f)} \left[\bar{\beta} - \sum_{i=1}^{n_s} \psi(\bar{q}(s_i; n_f, n_u)) [f(s_i|\bar{\beta}) - f(s_i|\underline{\beta})] |s_i - s_{i-1}| \right],$$

$$\bar{p}(c_j, n_f, n_u) = c_j + \bar{\beta} - \sum_{i=1}^{n_s} \psi(\bar{q}(s_i; n_f, n_u)) f(s_i|\bar{\beta}) |s_i - s_{i-1}|,$$

Third, given the equilibrium payment schedules, we calculate $\mathbb{E}(T|c_j, n_f, n_u)$ as follows:

$$\begin{aligned} & \mathbb{E}(T|c_j, n_f, n_u) \\ &= c_j + [1 - (1 - \pi_f(n_f))^{n_f}] [\underline{p}(c_j; n_f, n_u) - c_j] \\ &+ (1 - \pi_f(n_f))^{n_f} [1 - (1 - \pi_u(n_u))^{n_u}] \left[\bar{p}(c_j, n_f, n_u) - c_j + \sum_i^{n_s} \bar{q}(s_i; n_f, n_u) f(s_i|\bar{\beta}) |s_i - s_{i-1}| \right]. \end{aligned}$$

Therefore, the value of c_j does not affect the optimal choice of the extent of competition. The expected total cost of procurement with restrictions on entry can be calculated using equation (8) as follows:

$$V(\text{exclusive}, c_j, \lambda) = \sum_{n=1}^{n_b} \mathbb{E}(T|c_j, n, 0) \frac{(\lambda - 1)^{n-1} e^{-\lambda+1}}{(n-1)!} + \kappa\lambda - \delta,$$

As for the expected total cost without restrictions, $V(\text{nonexclusive}, c_j, \lambda)$, can be calculated

as follows, using based on equation (9).

$$\begin{aligned}
V(\text{nonexc.}, c_j, \lambda) &= \kappa\lambda - \delta + \sum_{n=1}^{n_b} \frac{(\lambda - 1)^{n-1} e^{-\lambda+1}}{(n-1)!} \left[\sum_{n_f=0}^n \mathbb{E}(T|c_j, n_f, n_u) \binom{n}{n_f} \rho(n)^{n_f} (1 - \rho(n))^{n_u} \right. \\
&\quad \left. + \sum_{n_f=0}^{n-1} \delta [1 - \pi_f(n_f)]^{n_f} [1 - (1 - \pi_u(n_u))^{n_u}] \binom{n}{n_f} \rho(n)^{n_f} (1 - \rho(n))^{n_u} \right],
\end{aligned}$$

where $n_u \equiv n - n_f$. We then solve λ_e and λ_o that minimize $V(\text{exclusive}, c_j, \lambda)$ and $V(\text{nonexclusive}, c_j, \lambda)$ respectively. Note that λ_e and λ_o do not depend on the value of c_j . Given the minimized costs, it is optimal for the procurer to impose a eligibility restriction if

$$V(\text{exclusive}, \lambda_e) - \eta \leq V(\text{nonexclusive}, \lambda_o).$$

Key Moments from the Equilibrium First, under the assumption that η is drawn from the standard logistic distribution, the probability that a contract is competed exclusively is given by:

$$\Pr(\text{exclusive}) = \frac{1}{1 + \exp(-V(\text{nonexc.}, \lambda_e) + V(\text{exclusive}, \lambda_o))}.$$

Second, the average number of bidders for nonexclusive and exclusive contracts, $\mathbb{E}(N_i|Y_i = 1)$ and $\mathbb{E}(N_i|Y_i = 0)$ respectively, is equal to λ_e and λ_o .

Third, the probability of having a fixed-price contract conditional on n participants for an exclusive competition, $\Pr(W_i = 1|N_i = n, Y_i = 1)$, is obtained directly from $\pi_f(n)$ for any $n = 1, \dots, n_b$, based on equation (4). $\Pr(W_i = 1|N_i = n, Y_i = 0)$ is based on the following equation:

$$\begin{aligned}
&\Pr(W_i = 1|N_i = n, Y_i = 0) \\
&= \sum_{j=0}^n \binom{n}{j} \rho(n)^j (1 - \rho(n))^{n-j} [1 - (1 - \pi_f(j))^j + (1 - \pi_f(j))^j (1 - (1 - \pi_u(n-j))^{n-j})].
\end{aligned}$$

Lastly, the expected value of the base and the final payments can be calculated based on the equilibrium contracts.